

MATH 251: Practice 25

July 14, 2015

Name: Solution

Use the Divergence Theorem to compute the flux of the vector field

$$\vec{F} = \langle x^2y + \sin(yz), xyz + 3z, e^y + z^2 \rangle$$

out of the rectangular prism $0 \leq x \leq 2$, $1 \leq y \leq 3$, $0 \leq z \leq 3$.

$$\nabla \cdot \vec{F} = 2xy + xz + 2z$$

$$\int_0^2 \int_0^3 \int_1^3 (2xy + xz + 2z) \, dy \, dz \, dx$$

$$= \int_0^2 \int_0^3 [xy^2 + xyz + 2yz] \Big|_1^3 \, dz \, dx$$

$$= \int_0^2 \int_0^3 (8x + 2xz + 4z) \, dz \, dx$$

$$= \int_0^2 (8xz + xz^2 + 2z^2) \Big|_0^3 \, dx$$

$$= \int_0^2 (24x + 9x + 18) \, dx$$

$$= \left[12x^2 + \frac{9}{2}x^2 + 18x \right]_0^2$$

$$= 48 + 18 + 36 = \boxed{102}$$