MATH 251: Practice 24

July 13, 2015

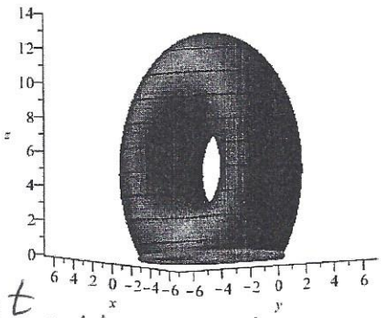
Name: Solution

Use Stokes' Theorem to evaluate the integral

$$\iint_S \text{curl}(\vec{F}) \cdot d\vec{S}$$

for the surface $\mathcal{S}$ with outward normal vector and vector field $\vec{F}$ below, where the boundary of $\mathcal{S}$ is the ellipse $4x^2 + y^2 = 16$ in the xy-plane. This boundary can be parametrized as $c(t) = \langle 2\cos(t), 4\sin(t), 0\rangle$, $0 \le t \le 2\pi$.

$\vec{F} = \langle 3x + 4yz, x + y + z, 3x^2 + 4y\rangle$



$F(c(t)) = \langle 6\cos t, 2\cos t + 4\sin t, 4\cos^2 t + 16\sin t)\rangle$

$c'(t) = \langle -2\sin(t), 4\cos(t), 0\rangle$

$$\oint_C \langle 6\cos(t), 2\cos(t) + 4\sin(t), 4\cos^2 t + 16\sin(t)\rangle \cdot \langle -2\sin(t), 4\cos(t), 0\rangle$$

$$= \int_0^{2\pi} -12\cos(t)\sin(t) + 8\cos^2 t + 16\cos(t)\sin(t)\, dt$$

$$= 8\left(\frac{\theta}{2} + \frac{\sin(2t)}{4}\right)\Big|_0^{2\pi} = 8\pi$$