

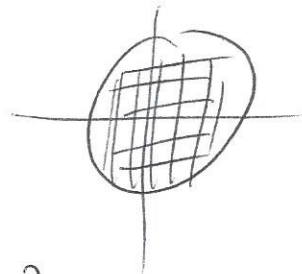
MATH 251: Practice 23

July 9, 2015

Name: Solutions

Use Green's Theorem to compute

$$\int_C (x^2y + e^x) dx + (xy^2 + y \sin(y)) dy$$



where C is the unit circle, oriented counterclockwise.

$$\int_C \overset{F_1}{(x^2y + e^x)} dx + \overset{F_2}{(xy^2 + y \sin(y))} dy$$

$$= \iint_D y^2 - x^2 dA$$

$$= \int_0^{2\pi} \int_0^1 (r^2 \sin^2 \theta - r^2 \cos^2 \theta) r dr d\theta$$

$$= \int_0^{2\pi} \int_0^1 r^3 \sin^2 \theta - r^3 \cos^2 \theta dr d\theta$$

$$= \int_0^{2\pi} \frac{1}{4} (\sin^2 \theta - \cos^2 \theta) d\theta$$

$$= -\frac{1}{4} \int_0^{2\pi} \cos 2\theta d\theta = \boxed{0}$$