MATH 251: Practice 22

July 8, 2015

Name: Solutions

Let $\mathcal{S} = G(u,v) = (u^2, v, u+v)$. $1 \le u \le 2$, $1 \le v \le 2$.

1. Find the normal vector to $\mathcal{S}$ as a function of u and v.

$T_u = \langle 2u, 0, 1 \rangle$

$T_v = \langle 0, 1, 1 \rangle$

$\vec{n} = \langle -1, -2u, 2u \rangle$

2. Compute the integrals

(a) $\iint_S z - y \, dS$ $\quad$ (b) $\iint_S \langle x, y, 1 \rangle \cdot d\vec{S}$

$\|\vec{n}\| = \sqrt{8u^2+1}$

(a) $= \int_1^2 \int_1^2 u\sqrt{8u^2+1} \, du\, dv$

$w = 8u^2+1$

$dw = 16u\, du$

$= \frac{1}{16} \int_1^2 \int_9^{33} w^{1/2} \, dw \, dv$

$= \frac{1}{24} \int_1^2 (33)^{3/2} - 9^{3/2} \, dv$

$= \boxed{\frac{1}{24}(33^{3/2} - 27)}$

(b) $\int_1^2 \int_1^2 \langle u^2, v, 1 \rangle \cdot \langle -1, -2u, 2u \rangle$

$= \int_1^2 \int_1^2 -u^2 - 2uv + 2u \, du\, dv$

$= \int_1^2 -\frac{u^3}{3} - u^2 v + u^2 \Big|_1^2 \, dv$

$= \int_1^2 -\frac{7}{3} - 3v + 3 \, dv$

$= \int_1^2 \frac{2}{3} - 3v \, dv$

$= \frac{2}{3}v - \frac{3}{2}v^2$

$= \frac{2}{3} - \frac{9}{2} = \boxed{-23/6}$