MATH 251: Practice 21

July 2, 2015

Name: Solutions

1. Compute the vector line integral of $\vec{F} = \langle y, x, z\rangle$ along the curve $\vec{c}(t) = \langle t, t^2, t^3\rangle$ from $t = 0$ to $t = 2$.

(a) $V = xy + z^2/2$. $\quad \int_{\vec{c}} \vec{F} \cdot d\vec{s} = V(2,4,8) - V(0,0,0)$

$= 8 + \frac{64}{2} = \boxed{40.}$

(b) $\vec{c}\,'(t) = \langle 1, 2t, 3t^2\rangle$. $\quad \vec{F}(\vec{c}(t)) = \langle t^2, t, t^3\rangle$

$$\int_{\vec{c}} \vec{F} \cdot d\vec{s} = \int_0^2 \langle t^2, t, t^3\rangle \cdot \langle 1, 2t, 3t^2\rangle \, dt$$

$$= \int_0^2 3t^2 + 3t^5 \, dt = t^3 + \frac{t^6}{2}\Big|_0^2 = 8 + \frac{64}{2} = \boxed{40}$$

2. Find a potential function for $\vec{F} = \langle 2xy + \sin(z), x^2 + e^y, x\cos(z) + z^2\rangle$, if it exists.

$\frac{\partial V}{\partial x} = 2xy + \sin z \longrightarrow V = x^2y + x\sin z + f_1(y,z)$

$\frac{\partial V}{\partial y} = x^2 + e^y \longrightarrow V = x^2y + e^y + f_2(x,z)$

$\frac{\partial V}{\partial z} = x\cos(z) + z^2 \longrightarrow V = x\sin(z) + \frac{z^3}{3} + f_3(x,y)$

$$\Longrightarrow \boxed{V = x^2y + x\sin(z) + e^y + \frac{z^3}{3} + C}$$