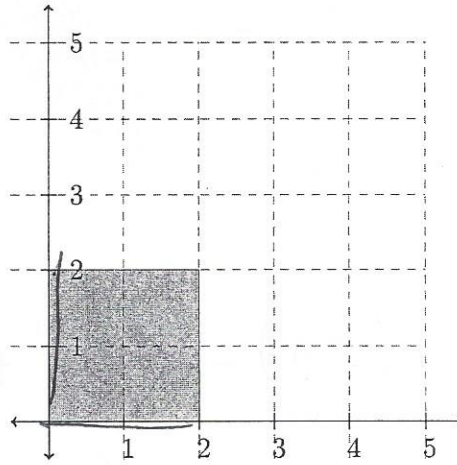
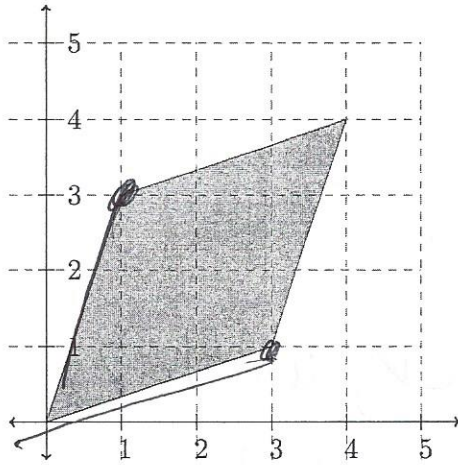


MATH 251: Practice 19

June 30, 2015

Name: Solutions

1. Find the change of variables going between the two regions below.



u from $0 \rightarrow 2$

$x : 0 \rightarrow 3$

$y : 0 \rightarrow 1$

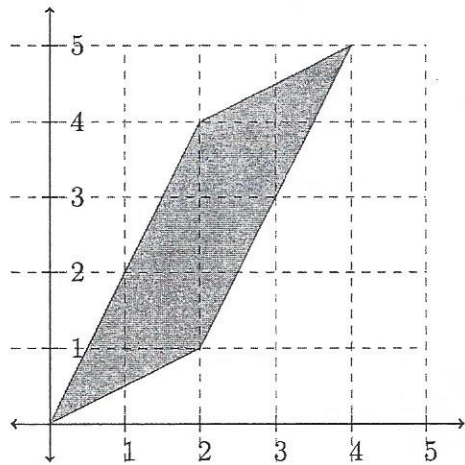
v from $0 \rightarrow 2$

$x : 0 \rightarrow 7$

$y : 0 \rightarrow 3$

$$G(u,v) = \left(\frac{3}{2}u + \frac{1}{2}v, \frac{1}{2}u + \frac{3}{2}v \right)$$

2. Integrate $f(x, y) = x^2 + y^2$ over the region drawn below.



$$G(u, v) = \langle 2u + 2v, u + 4v \rangle$$

$$\text{Jac}(G) = \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 6$$

$$\iint_D x^2 + y^2 \, dA = \int_0^1 \int_0^1 (2u + 2v)^2 + (u + 4v)^2 \cdot 6 \, du \, dv$$

$$= 6 \int_0^1 \int_0^1 (4u^2 + 8uv + 4v^2 + u^2 + 8uv + 16v^2) \, du \, dv$$

$$= 6 \int_0^1 \int_0^1 (5u^2 + 16uv + 20v^2) \, du \, dv$$

$$= 6 \int_0^1 \left[\frac{5}{3} u^3 + 8uv^2 + 20v^2 u \right]_0^1 \, dv$$

$$= 6 \int_0^1 \left(\frac{5}{3} + 8v + 20v^2 \right) \, dv$$

$$= 6 \left[\frac{5}{3} v + 4v^2 + \frac{20}{3} v^3 \right]_0^1 = \frac{25}{3} + 4 + \frac{60}{3} = \frac{77}{3} = 77$$