

# MATH 251: Practice 18

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Name: \_\_\_\_\_

Integrate  $f(x, y, z) = z$  over the region above the cone  $z^2 = x^2 + y^2$  and inside the sphere of radius 2,  $x^2 + y^2 + z^2 = 4$ .

- (a) Convert the boundary surfaces,  $z^2 = x^2 + y^2$  and  $x^2 + y^2 + z^2 = 4$ , and  $f(x, y, z) = z$  into spherical coordinates.
- (b) Use the boundaries to set up bounds on  $\rho$ ,  $\theta$ , and  $\phi$ .
- (c) Evaluate the integral in spherical coordinates.

For reference, the integral in rectangular coordinates is

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z \, dz \, dy \, dx$$