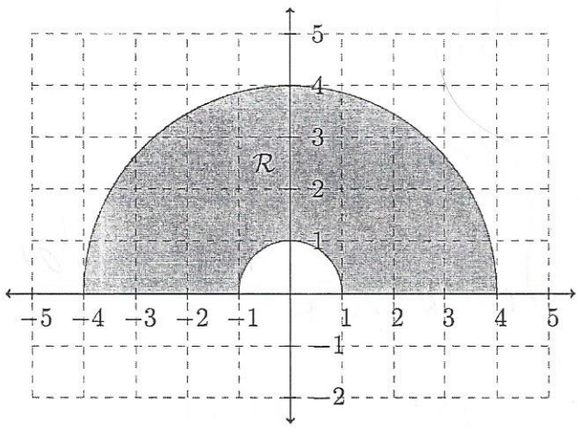
MATH 251: Practice 17

June 25, 2015

Name: Solutions

1. Integrate $f(x, y) = y$ over the region below.



$0 \le \theta \le \pi$

$1 \le r \le 4$

$$\int_0^{\pi}\int_1^4 r\sin\theta \cdot r\,dr\,d\theta$$

$$= \int_0^{\pi}\int_1^4 r^2\sin\theta\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{r^3}{3}\Big|_1^4 \sin\theta\,d\theta$$

$$= \frac{64-1}{3}\int_0^{\pi}\sin\theta\,d\theta$$

$$= 21(-\cos\theta)\Big|_0^{\pi} = \boxed{42}$$

$0 \le \theta \le \pi/2$. $1 \le r \le 2$.

2. Find the volume of the region in the first octant bounded by the cylindrical shells $1 \le x^2+y^2 \le 4$ and the planes $z = x + y$ and $z = 2x + 2y$.

$$\int_0^{\pi/2}\int_1^2 \int_{x+y}^{2x+2y} 1 \, dV.$$

$$\int_0^{\pi/2}\int_1^2 \int_{r\cos\theta + r\sin\theta}^{2r\cos\theta + 2r\sin\theta} 1 \, r \, dz \, dr \, d\theta$$

$$= \int_0^{\pi/2}\int_1^2 rz \Big|_{r\cos\theta + r\sin\theta}^{2r\cos\theta + 2r\sin\theta} dr \, d\theta.$$

$$= \int_0^{\pi/2}\int_1^2 r(r\cos\theta + r\sin\theta)\, dr \, d\theta$$

$$= \int_0^{\pi/2}\int_1^2 r^2\cos\theta + r^2\sin\theta \, dr \, d\theta$$

$$= \int_0^{\pi/2} \frac{r^3}{3}\Big|_1^2 (\cos\theta + \sin\theta)\, d\theta.$$

$$= \frac{7}{3}\left(\sin\theta - \cos\theta\right)\Big|_0^{\pi/2} = \boxed{14/3}.$$