MATH 251: Practice 15

June 23, 2015

Name: Solutions

1. Find the volume of the region in the first octant bounded by the planes $x = 4$, $y = 3$, and $2x - y + z = 10$.

$$\int_0^4 \int_0^3 \int_0^{10-2x+y} 1 \, dz \, dy \, dx$$

$$= \int_0^4 \int_0^3 10 - 2x + y \, dy \, dx$$

$$= \int_0^4 10y - 2xy + \frac{y^2}{2} \Big|_0^3 \, dx = \int_0^4 30 - 6x + \frac{9}{2}$$

$$= 30x - 3x^2 + \frac{9}{2}x \Big|_0^4 = 120 - 48 + 18 = \boxed{90}$$

2. Find $\iiint y \, dV$ over the region between the two paraboloids

$$z = 4 - 2x^2 - y^2 \qquad \text{and} \qquad z = x^2 + y^2$$

Hint: Solve for y as a function of x for the overlapping region, then make y your second integral.

$$4 - 2x^2 - y^2 = x^2 + y^2 \rightarrow 4 = 3y^2 + 2y^2. \quad y = \sqrt{2 - \tfrac{3}{2}x^2}$$

$$\iiint y \, dV = \iint_D \int_{x^2+y^2}^{4-2x^2-y^2} y \, dz \, dA = \iint_D (4 - 2x^2 - y^2)y - (x^2+y^2)y \, dA$$

$$= \iint_D 4y - 3x^2y - 2y^3 \, dA$$

$$= \int_0^{2/\sqrt{3}} \int_0^{\sqrt{2-3/2x^2}} 4y - 3x^2y - 2y^3 \, dy \, dx = \int_0^{2/\sqrt{3}} 2y^2 - \frac{3}{2}x^2y^2 - \frac{1}{2}y^4 \Big|_0^{\sqrt{2-3/2x^2}} dx$$

$$= \int_0^{2/\sqrt{3}} 2(2 - 3/2x^2) - 3/2x^2(2 - 3/2x^2) - \frac{1}{2}(2 - 3/2x^2)^2 = \frac{1}{2}(2 - 3/2x^2)^2$$

$$= \frac{1}{2}\int_0^{2/\sqrt{3}} 4 - 6x^2 + 9/4x^4 \, dx = \frac{1}{2}\left(4x - 2x^3 + \frac{9}{20}x^5\right)\Big|_0^{2/\sqrt{3}}$$

$$= \frac{1}{2}\left(8/\sqrt{3} - \frac{16}{3\sqrt{3}} + \frac{9}{20} \cdot \frac{32}{9\sqrt{3}}\right.$$

$$\frac{1}{2}\left(\frac{8}{\sqrt{3}} - \frac{16}{3\sqrt{3}} + \frac{32}{20}\frac{1}{\sqrt{3}}\right)$$

$$\frac{1}{2}\left(\frac{1}{\sqrt{3}}\left(8 - \frac{16}{3} + \frac{\cancel{32}}{\cancel{20}}\frac{8}{5}\right)\right)$$

$$= \frac{1}{2}\frac{8}{\sqrt{3}}\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{8}{\sqrt{3}} \cdot \frac{8}{15} = \cancel{\frac{64}{45}}$$

$$= \frac{\cancel{64}\ 32}{15\sqrt{3}}$$