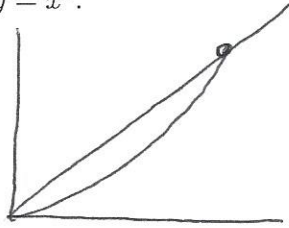


MATH 251: Practice 14

June 22, 2015

Name: Solutions

1. Compute the integral of $f(x, y) = x + y$ over the region in the first quadrant between the graphs of $y = x$ and $y = x^2$.



$$\int_0^1 \int_{x^2}^x x+y \, dy \, dx$$

$$= \int_0^1 \left. xy + \frac{y^2}{2} \right|_{y=x^2}^{y=x} dx$$

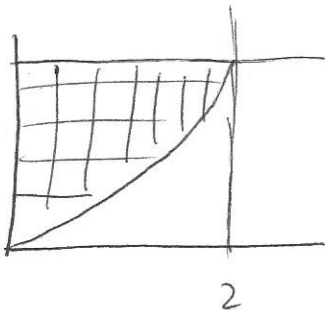
$$= \int_0^1 x^2 + \frac{x^2}{2} - x^3 - \frac{x^4}{2} dx$$

$$= \int_0^1 \left(\frac{3}{2}x^2 - x^3 - \frac{x^4}{2} \right) dx = \left. \frac{x^3}{2} - \frac{x^4}{4} - \frac{x^5}{10} \right|_0^1$$

$$= \frac{1}{2} - \frac{1}{4} - \frac{1}{10} = \frac{1}{4} - \frac{1}{10} = \boxed{\frac{3}{20}}$$

2. Change the order of integration, then compute the integral

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$$\int_0^2 \int_{x^2}^4 x e^{y^2} \, dy \, dx$$

$$\int_0^4 \int_0^{\sqrt{y}} x e^{y^2} \, dx \, dy$$

$$\frac{1}{2} \int_0^4 x^2 e^{y^2} \Big|_0^{\sqrt{y}} dy = \frac{1}{2} \int_0^4 y e^{y^2} dy$$

$$u = y^2$$

$$du = 2y \, dy$$

$$= \frac{1}{4} \int_0^{16} e^u \, du = \frac{1}{4} e^u \Big|_0^{16}$$

$$= \boxed{\frac{1}{4} (e^{16} - 1)}$$