

MATH 251: Practice 13

June 18, 2015

Name: Solutions

Compute the following integrals.

$$\iint_{\mathcal{R}} 2x + 3y^2 \, dA \quad \text{over } \mathcal{R} = [0, 3] \times [1, 2]$$

$$\int_0^2 \int_0^1 x \sin(xy) \, dx \, dy$$

$$\int_1^2 \int_0^3 2x + 3y^2 \, dx \, dy = \int_1^2 \left. x^2 + 3y^2 x \right|_{x=0}^{x=3} dy$$

$$= \int_1^2 9 + 9y^2 \, dy$$

$$= 9y + 3y^3 \Big|_1^2$$

$$= 18 + 24 - 9 - 3 = \boxed{30}$$

Switch order

$$\int_0^1 \int_0^2 x \sin(xy) \, dy \, dx$$

$$u = xy$$

$$du = x \, dy$$

$$= \int_0^1 \left(\int \sin u \, du \right) dx = \int_0^1 -\cos(xy) \Big|_{y=0}^{y=2} dx$$

$$= \int_0^1 1 - \cos(2x) \, dx = \left. x - \frac{1}{2} \sin(2x) \right|_0^1$$

$$= \boxed{1 - \frac{1}{2} \sin(2)}$$