MATH 251: Practice 12

June 17, 2015

Name: Solutions

Find the maximum value of $2x + 3y - z$ on the ellipsoid $x^2 + 3y^2 + 2z^2 = 4$

$$f(x,y,z) = 2x + 3y - z$$

$$g(x,y,z) = x^2 + 3y^2 + 2z^2 - 4 = 0$$

$$\nabla f = \langle 2, 3, -1 \rangle$$

$$\nabla g = \langle 2x, 6y, 4z \rangle$$

$$\nabla f = \lambda \nabla g.$$

$$2 = 2\lambda x \quad \rightarrow \quad x = \frac{1}{\lambda}$$

$$3 = 6\lambda y \qquad y = \frac{1}{2\lambda}$$

$$-1 = 4\lambda z. \qquad z = \frac{-1}{4\lambda}$$

Plug into g

$$\frac{1}{\lambda^2} + \frac{3}{4\lambda^2} + \frac{1}{8\lambda^2} = 4.$$

$$4\lambda^2 = 1 + \frac{3}{4} + \frac{1}{8} = \frac{15}{8}$$

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$$\lambda^2 = \frac{15}{32} \qquad \lambda = \pm\sqrt{\frac{15}{32}} = \pm\frac{1}{4}\sqrt{15/2}.$$

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Find the maximum value of $2x + 3y - z$ on the ellipsoid $x^2 + 3y^2 + 2z^2 = 4$

$$X = \frac{1}{\lambda} = \pm 4\sqrt{\frac{2}{15}}$$

$$y = \frac{1}{2\lambda} = \pm 2\sqrt{\frac{2}{15}}$$

$$z = \frac{-1}{4\lambda} = \mp\sqrt{\frac{2}{15}}$$

For +

$$2x+3y-z = 8\sqrt{\frac{2}{15}} + 6\sqrt{\frac{2}{15}} + 1\sqrt{\frac{2}{15}} = 15\sqrt{\frac{2}{15}} = \sqrt{30}$$

For − Some work $\rightarrow$ $-\sqrt{30}$

Max Value