

MATH 251: Practice 11

June 16, 2015

Name: Solutions

1. Find  $\frac{\partial f}{\partial s}$  in terms of  $s$  and  $t$  if  $f(x, y) = e^x + 2xy$  and

$$x = s^2 + t^2 \quad y = 2t + s.$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s}$$

$$= (e^x + 2y)(2s) + (2x)(1)$$

$$= \left( e^{(s^2+t^2)} + 2(2t+s) \right) 2s + 2(s^2+t^2)$$

$$= 2se^{s^2+t^2} + 8st + 6s^2 + 2t^2$$

2. Find and classify all local extreme values of  $f(x, y) = \frac{x^3}{3} - 3x + y^2 - 2xy$ .

$$f_x = x^2 - 3 - 2y$$

$$f_y = 2y - 2x = 0$$

$$= x^2 - 2x - 3 = 0$$

$$\Rightarrow x = y$$

$$(x-3)(x+1) = 0$$

$$(P: (3, 3) \quad (-1, -1).$$

$$x = 3 \text{ or } x = -1$$

$$f_{xx} = 2x$$

$$f_{xy} = -2$$

$$f_{yy} = 2.$$

$$D = f_{xx}f_{yy} - f_{xy}^2$$

$$= 4x - 4$$

$$(3, 3) \rightarrow D = 12 - 4 = 8 > 0 \quad f_{yy} > 0 \Rightarrow \text{Min}$$

$$(-1, -1) \quad D = -4 - 4 = -8 < 0 \Rightarrow \text{Saddle.}$$