Systems of Differential Equations

The next object that we want to talk about are systems of differential equations. These occur very frequently in physical situations, because it is almost impossible for only one quantity to be changing in a physical system.

- Multiple things changing lead us to systems.

Ex

- Predator-Prey dynamics
- Multiple Tanks
- Chemical Reactors

- Can be any order
- Any number of components.

First Order Systems

What is the idea of first order systems?

→ Only the first derivative shows up.

Two unknown functions $x(t)$, $y(t)$

$$\left.\begin{aligned} \frac{dx}{dt} &= f(t, x, y) \\ \frac{dy}{dt} &= g(t, x, y) \end{aligned}\right] \quad \vec{v}\,' = \vec{F}(t, \vec{v})$$

$$\vec{F} = \begin{bmatrix} f(t, \vec{v}) \\ g(t, \vec{v}) \end{bmatrix}$$

$$v(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

<u>n functions</u>

$$\left.\begin{aligned} \frac{dx_1}{dt} &= f_1(t, x_1, \ldots, x_n) \\ &\vdots \\ \frac{dx_n}{dt} &= f_n(t, x_1, \ldots, x_n) \end{aligned}\right] \quad \vec{x}\,' = \vec{F}(t, \vec{x})$$

$$\vec{F} = \begin{bmatrix} f_1(t, \vec{x}) \\ \vdots \\ f_n(t, \vec{x}) \end{bmatrix}$$

First Order Linear Systems

What does it look like if this system is linear?

$$\frac{dx}{dt} = f(t,x,y) = a(t)x + b(t)y + f_1(t)$$

$$\frac{dy}{dt} = g(t,x,y) = c(t)x + d(t)y + f_2(t)$$

$$\begin{cases} \dfrac{dx_1}{dt} = \underline{a(t)x_1} + \underline{b(t)x_2} + f_1(t) \\ \dfrac{dx_2}{dt} = c(t)x_1 + d(t)x_2 + f_2(t) \end{cases}$$

$$\vec{x}' = \begin{bmatrix} a(t)x_1 + b(t)x_2 \\ c(t)x_1 + d(t)x_2 \end{bmatrix} + \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$\vec{x}' = \begin{bmatrix} a(t) & b(t) \\ c(t) & d(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$\vec{x}' = A(t)\vec{x} + \vec{f}$$