Systems of ODEs

Now, we're going to tie everything back to differential equations, and eventually work in all of the linear algebra stuff that we just covered.

Definition 0.1. A *first order system of differential equations* is a set of n equations involving n unknown functions and their first derivatives.

$$\frac{dx_1}{dt} = f_1(t, x_1, x_2, \ldots, x_n)$$

$$\vdots$$

$$\frac{dx_n}{dt} = f_n(t, x_1, x_2, \ldots, x_n)$$

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} \qquad \vec{x}\,'(t) = \begin{bmatrix} \frac{dx_1}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{bmatrix}$$

$$\boxed{\vec{x}\,' = \vec{F}(t, \vec{x})} \qquad \vec{F} = \begin{bmatrix} f_1(t, x_1, \ldots, x_n) \\ \vdots \\ f_n(t, x_1, \ldots, x_n) \end{bmatrix}$$

Solutions to Systems

What does the solution to a system of ODEs look like?

In vector form, it is a vector valued function $\boxed{\vec{x}(t)}$ that will satisfy the equation when I plug it in.

$$\frac{dx_1}{dt} = f_1(t, x_1, x_2, x_3)$$

→ Need to check all components together.

→ Must solve ALL Equations.

→ They don't have to be linear, but a lot of examples will be.

Example. Show that

$$\boxed{\vec{x}(t) = \begin{bmatrix} 2e^{-2t} + 4e^{3t} \\ e^{3t} - 2e^{-2t} \end{bmatrix}} \rightarrow \begin{bmatrix} 2+4 \\ 1-2 \end{bmatrix}$$

solves the system

$$\frac{dx_1}{dt} = 2x_1 + 4x_2 \qquad \frac{dx_2}{dt} = x_1 - x_2 \qquad \vec{x}(0) = \begin{bmatrix} 6 \\ -1 \end{bmatrix}.$$

$$x_1(t) = 2e^{-2t} + 4e^{3t}$$

$$x_2(t) = e^{3t} - 2e^{-2t}$$

$$\frac{dx_1}{dt} = -4e^{-2t} + 12e^{3t}$$

$$2x_1 + 4x_2 = 2(2e^{-2t} + 4e^{3t}) + 4(e^{3t} - 2e^{-2t})$$

$$-4e^{-2t} + 12e^{3t}$$

$$\frac{dx_2}{dt} = 3e^{3t} + 4e^{-2t}$$

$$x_1 - x_2 = 2e^{-2t} + 4e^{3t} - (e^{3t} - 2e^{-2t})$$