Undetermined Coefficients

The method of undetermined coefficients works very similar to how it worked for second order equations.

- Guess the form of the solution based on the non-homogeneous part
- Solve for these "unknown coefficients" to get the solution.
  - → All of the unknowns will be vectors (so more constants)
  - → Process is the same.
- Multiply by t if it solves the homogeneous problem
  - → Include both t and non-t version.

$$\vec{A}te^t + \vec{B}e^t$$

$$\vec{x}' = A(t)\vec{x} + \vec{f}$$

Variation of Parameters

Variation of parameters looks similar as well, but is even easier to state in the case of systems.

Find general solution to homogeneous problem. $\vec{x}' = A(t)\vec{x}$ which is

$$c_1\vec{x}_1(t) + c_2\vec{x}_2(t) + \dots c_n\vec{x}_n(t)$$

General solution: $X(t)\vec{c}$

$$X = [\vec{x}_1 | \vec{x}_2 | \vec{x}_3 | \dots \vec{x}_n]$$

Fundamental Matrix of solutions.

Guess $\vec{x} = X(t)\vec{u}(t)$

$$X'(t)\vec{u}(t) + X(t)\vec{u}'(t) = \vec{x}'(t) = A(t)X(t)\vec{u}(t) + \vec{f}(t)$$

$$X'(t) = A(t)X(t)$$

$$X(t)\vec{u}'(t) = \vec{f}(t)$$

$$\vec{u}'(t) = X(t)^{-1}\vec{f}(t)$$

$$\vec{u}(t) = \int X(t)^{-1}\vec{f}(t)\,dt$$

Non Homogeneous Solution

$$\boxed{\vec{x}(t) = X(t)\int X(t)^{-1}\vec{f}(t)\,dt}$$