Linear Systems

A lot of the terminology from higher order linear differential equation carry over to linear systems.

$$\vec{x}' = \vec{F}(t, \vec{x})$$

$$\vec{x}(t_0) = \begin{bmatrix} x_{10} \\ x_{20} \\ \vdots \\ x_{n0} \end{bmatrix}$$

$$\boxed{\vec{x}' = A(t)\vec{x} + \vec{G}(t)}$$

$$\frac{dx_1}{dt} = a_{11}(t)x_1 + a_{12}(t)x_2 + \cdots + a_{1n}(t)x_n + g_1(t)$$

$\vec{G}(t) = \vec{0}$, Homogeneous System

$\vec{G}(t) \neq \vec{0}$, Non-Homogeneous

For a general solution to Homogeneous, we want to be able to meet every initial condition.

How do we know if we have enough solutions go justify calling it a "general solution"? This is where we need linear algebra.

$\vec{x}' = A(t)\vec{x}$ $n \times n$ $\vec{x}(t_0) =$ vector with n components.

General Solution

$$c_1\vec{x}_1(t) + c_2\vec{x}_2(t) + \dots + c_k\vec{x}_k(t)$$

At t_0

$$c_1\underbrace{\vec{x}_1(t_0)} + c_2\underbrace{\vec{x}_2(t_0)} + \dots + c_k\underbrace{\vec{x}_k(t_0)}$$

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k$$

Can this meet every inital condition for an appropriate choice of $c_1, \dots, c_k$?

Need $\text{span}\{\vec{v}_1, \dots \vec{v}_k\} = \mathbb{R}^n$

$\rightarrow$ Need $\vec{v}_1, \dots, \vec{v}_k$ to be linearly independent

$\rightarrow$ span $\mathbb{R}^n$ $\rightarrow$ Need n of them.