First-Order Equations and Slope Fields

The first type of equation we will be focusing on is first-order equations. With only one derivative involved, these *should* be the easiest ones to solve.

Assume We can always solve out for the highest order derivative.

$$\frac{dy}{dt} = f(t, y)$$

How can we visualize this equation?

$$\frac{dy}{dt} = f(t,y)$$

Can evaluate at any point.

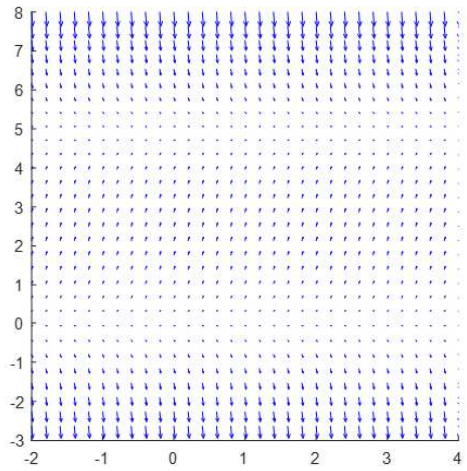
This is the slope of the tangent line to $y(t)$, the solution.

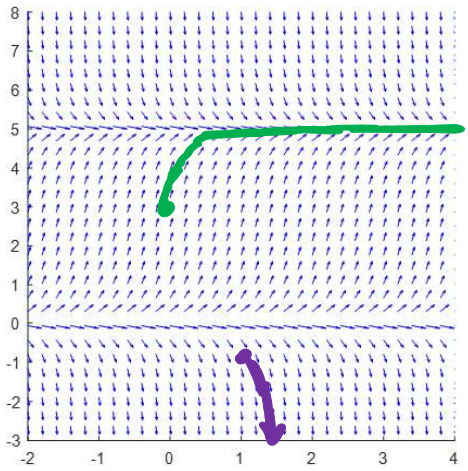
A **slope field** is a drawing where small line segments are drawn to indicate the slope of the tangent line to a solution curve.

→ I can follow the slopes to get an idea of what the solution does over time.

Example. Consider the slope field pictured below. What does the solution look like with $y(0) = 3$? What about with $y(1) = -1$?

All rescaled





For $y(0)=3$, the solution approaches 5

For $y(1)=-1$, the solution goes to $-\infty$.