Separable Equations

Now, we want to actually try to solve these equations. The first type we want to look at are ones that can be solved by direct integration. We've seen some examples of this already.

$$\boxed{\frac{dy}{dt} = t^2}$$

$$\int \frac{dy}{dt}\, dt = y(t)$$

Fundamental Theorem of Calculus

$$\int t^2 dt = \frac{t^3}{3} + C$$

This will work for anything $\frac{dy}{dt} = h(t)$

→ Integrate both sides in t.

So, we can solve equations this way if the right-hand side is only a function of t. What about more complicated functions? This comes back to the idea of the differential from Calculus 1.

→ You need a differential to compute an integral.

If we have $y(t)$, then dy is defined as

$$dy = y'(t)\,dt$$

$$dy = \boxed{\frac{dy}{dt}}\,dt$$

$$\frac{dy}{dt} = t^2 \rightarrow \int dy = \int t^2\,dt$$

$$\boxed{y = \frac{t^3}{3} + C}$$

Definition. A differential equation $\frac{dy}{dt} = f(t, y)$ is said to be *separable* if it

can be written as

$$g(y)\frac{dy}{dt} = h(t)$$

$$g(y)\,dy = h(t)\,dt$$

- Once the equation is in this form, I can integrate both sides.
  - → Only need 1 $+C$ to make it work. Can put it on either side.
- May not be an explicit solution $y(t) = \ldots$, but that's fine.

Example. Find the general solution of the differential equation $\frac{dy}{dt} = yt$.

$$\frac{1}{y}\frac{dy}{dt} = t$$

$$\int \frac{1}{y}\,dy = \int t\,dt$$

$$\ln|y| = \frac{t^2}{2} + C$$

$$\boxed{y = e^{t^2/2 + C} = De^{t^2/2}}$$

for $D = e^C$

$$\frac{dy}{dt} = \frac{2t}{2}\cdot De^{t^2/2} = ty \quad \checkmark$$