

Variation of Parameters

A second method that can be used to solve non-homogeneous equations is variation of parameters. This is a method that works in general for all second-order linear equations, but does not always result in answers that can be found explicitly.

Assume that we have the equation

$$y'' + p(t)y' + q(t)y = g(t)$$

and we know that the general solution to the homogeneous equation is $C_1y_1(t) + C_2y_2(t)$. To solve the non-homogeneous problem, we set $y = u_1y_1 + u_2y_2$ and plug it in.

The solution

From here, we can figure out what u_1 and u_2 should be.