

Undetermined Coefficients

Based on the results before, we know that the general solution to a non-homogeneous equation

$$y'' + p(t)y' + q(t)y = g(t)$$

is

$$y_p(t) + C_1y_1(t) + C_2y_2(t)$$

where y_p is any solution of the non-homogeneous problem and y_1 and y_2 are the linearly independent solutions of the corresponding homogeneous problem. In the case of constant-coefficient equations, we know how to get y_1 and y_2 . Then we just need to find y_p .

The first method we have for finding this solution is the “Method of Undetermined Coefficients.” Let’s look at this method through an example.

Example. Find the general solution to the differential equation

$$y'' + 3y' - 4y = 3e^{2t}$$

The main point here that helped us solve this problem was that exponential functions repeat themselves when we take derivatives, and that we had a constant coefficient equation. What other functions do this?