Undamped Forced Oscillators

What is different if there is no damping in the problem? Well, in that case, the transient solution isn't really transient. It doesn't go away because there is no exponential term.

$$ay'' + cy = F_0 \cos(\omega t)$$

Homogeneous Problem

$$ar^2 + c = 0$$

$$r = \pm\sqrt{-\frac{c}{a}} = \sqrt{\frac{c}{a}}\, i$$

General Solution

$$C_1 \cos\left(\sqrt{\tfrac{c}{a}}\, t\right) + C_2 \sin\left(\sqrt{\tfrac{c}{a}}\, t\right)$$

$$\omega_0 = \sqrt{\frac{c}{a}}$$ "Natural Frequency"

$$\boxed{C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t)}$$

→ Does not decay to 0 as t gets large.

Non-homogeneous

$$A \cos(\omega t)$$

$$-Aa\omega^2 \cos(\omega t) + cA \cos(\omega t) = F_0 \cos(\omega t)$$

$$F_0 = cA - Aa\omega^2$$

or

$$A = \frac{F_0}{c - a\omega^2} = \frac{F_0}{a(\omega_0^2 - \omega^2)}$$

Full general Solution

$$C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t) + \frac{F_0}{a(\omega_0^2 - \omega^2)} \cos(\omega t)$$

$\omega_0 \neq \omega$

- Nothing decays away.
  - → Long term interaction.
  - → Initial conditions Continue to matter.