Types of Second Order Equations

A second order equation is one where the highest derivative that shows up in the equation is the second derivative. This means that the most general second order equation is of the form

$$y'' = f(t, y, y')$$

where this f can be any function of 3 variables.

$$y'' = e^{y'} \cdot t^2 \sin(ty)$$

Unlike first order equations, there aren't really any general methods for solving these analytically.

→ We want to classify these equations so we know what we can and can't solve.

For this reason, we want to restrict the types of equations we consider and try to solve.

A linear second-order equation has the form

$$A(t)y'' + B(t)y' + C(t)y = F(t)$$

In standard form

$$\boxed{y'' + p(t)y' + q(t)y = g(t)}$$

- The equation is homogeneous if $g(t) = 0$, non-homogeneous otherwise.
- The equation is of constant-coefficient if the functions $p(t)$ and $q(t)$ are constants.

$$y'' + 4y' - 7y = e^t$$

- Constant Coefficient
- Non-Homogeneous.

These are in increasing order of what we can solve. We can solve all constant-coefficient homogeneous equations, and a lot of constant-coefficient non-homogeneous equations. Beyond that, there isn't too much hope.

No general method like integrating factors for first-order linear equations.

→ Only specific cases will work.

Focus on constant-coefficient homogeneous, then build to non-homogeneous.