Non-Homogeneous Linear Equations

Now, we want to look at non-homogeneous equations and think about how we would solve them. The most general form of a second-order, linear, non-homogeneous equation is

$$y'' + p(t)y' + q(t)y = g(t)$$

for functions $p(t)$, $q(t)$, and $g(t)$.

How can we try to solve these equations? At least for the constant-coefficient versions, we know how to solve the corresponding homogeneous equation. Can we get back to that equation somehow?

This work results in the following theorem:

Theorem 0.1. *Let $y_1(t)$ and $y_2(t)$ be two solutions to the non-homogeneous equation*

$$y'' + p(t)y' + q(t)y = g(t).$$

Then, the difference $y_1 - y_2$ solves the corresponding homogeneous equation

$$y'' + p(t)y' + q(t)y = 0.$$