

Modeling with Second Order Equations

Second order equations are also very useful in modeling situations. In general, this comes from Newton's Laws of Motion.

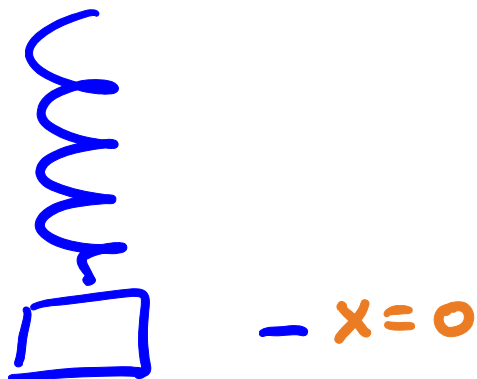
$$F = m a$$

Second derivative
of the position

depends
on position

$$5x = 2x''$$

The first main example of this is a mass on a spring.



$$m \ddot{x} = F_{\text{net}} = -kx$$

Hooke's Law

k = spring constant

$$- \gamma v \quad \text{---} \quad \dot{x}$$

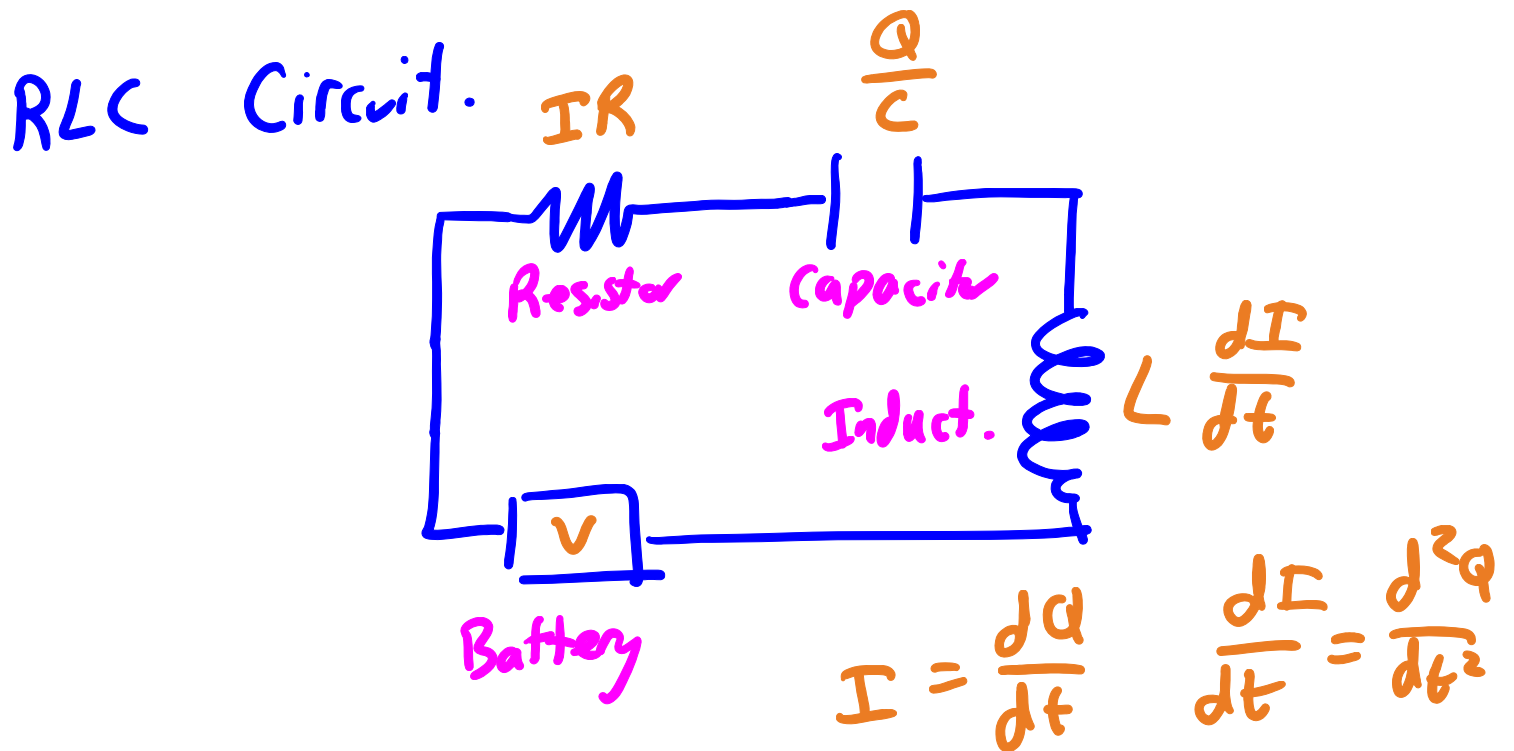
Drag Force

γ - drag coefficient.

$$m \ddot{x} = -kx - \gamma \dot{x}$$

A similar situation arises in circuits.

Kirchhoff's Law: Total voltage drop around a closed circuit must be zero.



$$V + IR + \frac{Q}{C} + L \frac{dI}{dt} = 0$$

$$L \cdot \frac{d^2q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} Q = -V$$