Modeling Non-Homogeneous Equations

Our original discussion of second-order equations started with physical problems.

Mass on a Spring

RLC Circuits

$$ay'' + by' + cy = g(t)$$

$g(t) = 0 \rightarrow$ Free vibrations

$g(t) \neq 0 \rightarrow$ Forced vibrations

$$F_{net} = ay'' = -cy - by' + \boxed{F(t)}$$

Applied Force

If $b = 0 \rightarrow$ undamped forced vibration

If $b > 0 \rightarrow$ damped forced vibration

Assume: $\boxed{g(t) = F_0 \cos(\omega t)}$

Damped Forced Oscillators

If we have a damped equation of the form

$$ay'' + by' + cy = F_0 \cos(\omega t),$$

there are three options for the solutions to the corresponding homogeneous equation:

- Real + Distinct → Both negative
- Complex roots → Real part negative
- Repeated roots → Root is negative

↳ All of these have exponential terms with negative powers.

All go to 0 as t gets large.

$$e^{-3t},\ e^{-2t}$$

$$e^{-4t}\sin(3t),\ e^{-4t}\cos(3t)$$

$$e^{-t},\ te^{-t}$$

For the non-homogeneous part, the solution is

$$A\cos(\omega t) + B\sin(\omega t)$$

Do not go to 0 as time goes on.

General Solution

$$\underbrace{C_1 y_1(t) + C_2 y_2(t)}_{} + \underbrace{A\cos(\omega t) + B\sin(\omega t)}_{}$$

- Solves Homog. eqn
- $\to 0$ as $t \to \infty$

"Transient Solution"

- Solves Non-homog
- $\not\to 0$ as $t \to \infty$

"Steady-State Solution"

"Steady periodic solution"

Note: Initial conditions affect C_1, C_2, which are in the transient solution.