Existence and Uniqueness Theorem

Just like for first order equations, we have an existence and uniqueness theorem for second-order linear equations.

Theorem. *Consider the initial value problem*

$$y'' + p(t)y' + q(t)y = g(t) \qquad y(t_0) = y_0,\ y'(t_0) = y'_0,$$

where p, q, and g are continuous functions on an open interval I containing t_0. This problem has exactly one solution $y = \phi(t)$, and this solution exists throughout the interval I.

Fundamental Set of Solutions

Now that we know solutions exist and are unique to these equations, we can talk about fundamental sets (or a basis) of solutions.

Definition. We say that y_1 and y_2 form a **fundamental set** of solutions to the differential equation $y'' + p(t)y' + q(t)y = 0$ if the set of all linear combinations $c_1y_1 + c_2y_2$ contains all solutions to the differential equation.

Extra Terminology