Applications of Non-Linear Systems

There are two main population type models that are formed and discussed in the context of non-linear systems. These are competing species models and predator-prey models.

Competing Species

- Two species which both grow on their own
- Negative interaction factor due to lack of resources.

$$\frac{dx}{dt} = ax - bxy \qquad \frac{dy}{dt} = cy - dxy$$

$$\frac{dx}{dx} = ax(K - x) - bxy \qquad \frac{dy}{dt} = cy(M - y) - dxy$$

Carrying Capacity

Predator-Prey interaction

Two species

Prey: • Grow on their own
• Interaction with predator is negative.

Predator: • Will die off on own
• Interaction with prey is positive

$$\frac{dx}{dt} = ax - bxy \qquad \frac{dy}{dt} = -dy + cxy$$

$$\frac{dx}{dt} = ax(K-x) - bxy$$

Example. Consider the system below. Find and classify all equilibrium solutions of the system.

$$x(3-x)-xy \qquad y(3-2y)-0.5xy$$

$$\frac{dx}{dt} = x(3-x-y) \qquad \frac{dy}{dt} = y(3-2y-0.5x)$$

Which of the two models does this fit? $\to$ Competing species

$y=3-x$

$x=0 \quad 3-x-y=0$

$y=0 \quad 3-2y-\frac{x}{2}=0$

$(0,0) \quad (0,3/2) \quad (3,0) \quad (2,1)$

$3-2(3-x)-\frac{x}{2}=0$

$3-6+2x-\frac{x}{2}=0$

$\frac{3}{2}x=3$

$x=2,\ y=1$

$F(x,y)=x(3-x-y)$

$G(x,y)=y(3-2y-0.5x)$

$F_x=(3-x-y)-x$

$F_y=-x$

$G_x=-0.5y$

$G_y=3-2y-0.5x-2y$

$$\begin{bmatrix} 3-2x-y & -x \\ -0.5y & 3-4y-0.5x \end{bmatrix}$$

$(0,0) \rightarrow \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Nodal Source

$\lambda = 3, 3$

$(0, 3/2) \rightarrow \begin{bmatrix} \underline{3/2} & 0 \\ -3/4 & \underline{-3} \end{bmatrix}$

Saddle

$\lambda = 3/2, -3$

$(3,0) \rightarrow \begin{bmatrix} -3 & -3 \\ 0 & 3/2 \end{bmatrix}$

Saddle

$\lambda = 3/2, -3$

$(2,1) \rightarrow \begin{bmatrix} -2 & -2 \\ -1/2 & -2 \end{bmatrix}$

Nodal sink

$\lambda = -1, -3$

$(-2-\lambda)(-2-\lambda) - 1$

$\lambda^2 + 4\lambda + 3 = (\lambda+3)(\lambda+1)$