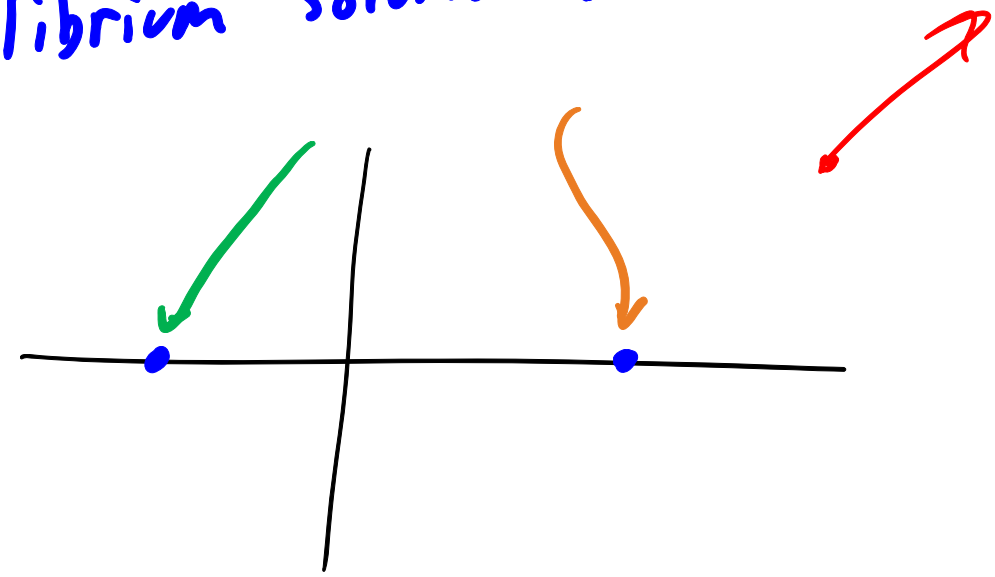
Behavior of Non-Linear Systems

Non-Linear systems and equations can have behavior that is not possible for their linear counterparts.

Ex Two different Nodal sink equilibrium solutions.



From Non-linear equations

→ Solutions stop existing

→ Non-uniqueness.

Why can this happen?

Non-linearity makes things a lot more complicated.

For a linear system there is only one equilibrium solution at $\vec{0}$

→ This drives behavior everywhere

For non-linear:

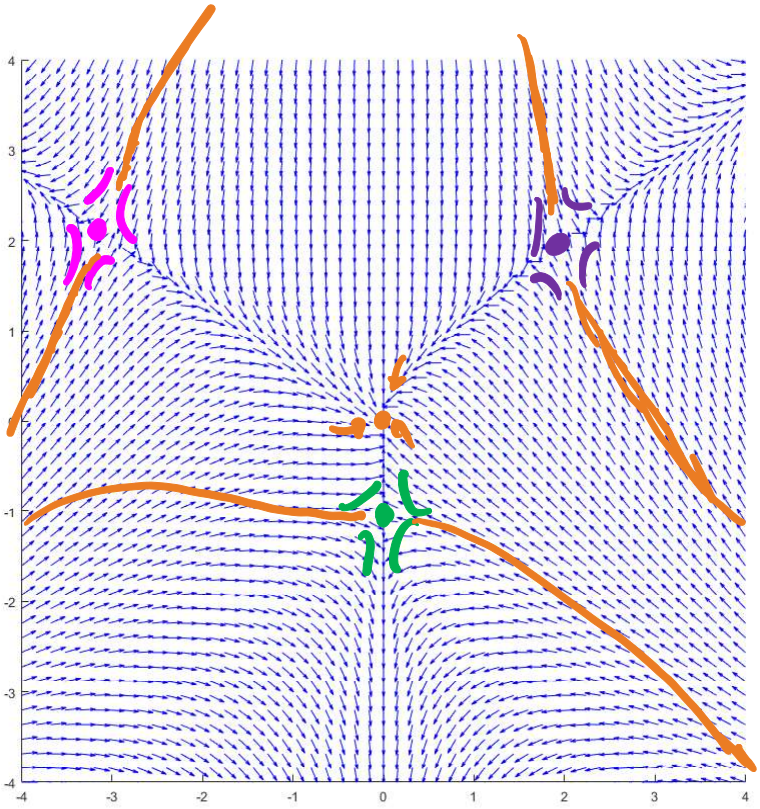
- Might be multiple eq. solutions
- Can get behavior that does not match the linear system.

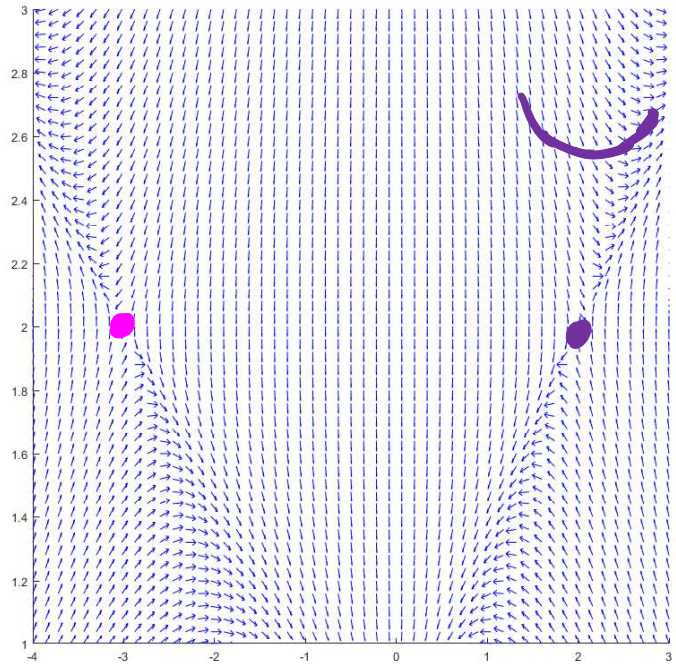
- In general, need to solve the problem to see what happens for a random initial condition.

Example. Consider the slope field drawn below for the system

$$\frac{dx}{dt} = x(y-2)$$
$$\frac{dy}{dt} = (x-y)(x+y+1)$$

What happens to solutions with a variety of initial conditions?




3
2.8
2.6
2.4
2.2
2
1.8
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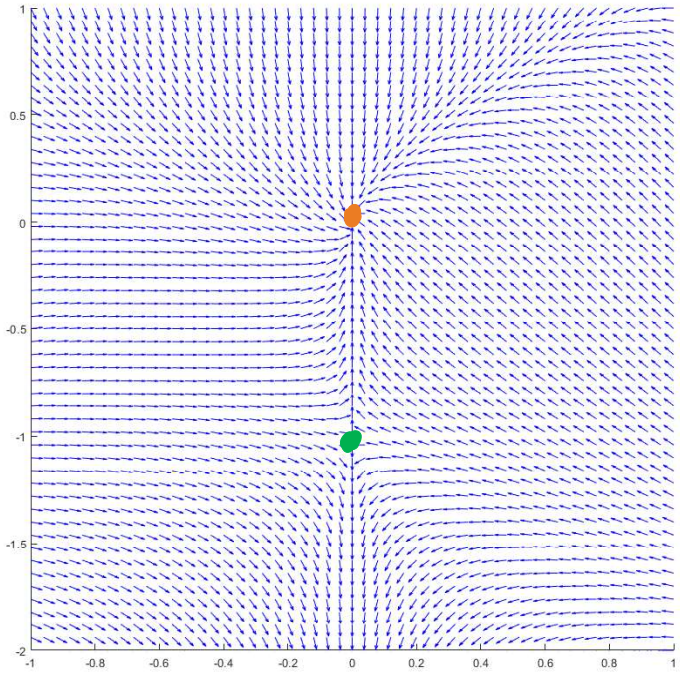
1
-4
-3
-2
-1
0
1
2
3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0
-0.5
-1
-1.5
-2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Terminology

How do we talk about these concepts?

Basin of Attraction

Set of all points where if I start a solution there, it will converge towards the sink.

Separatrix / Separatrices

Boundary curves that determine how the solution behaves.