Non-Linear Systems

It turns out that most systems of differential equations are not linear.

Linear Systems

$$\frac{dx}{dt} = a(t)x + b(t)y + f_1(t)$$

$$\frac{dy}{dt} = c(t)x + d(t)y + f_2(t)$$

Most physical systems involve direct interaction terms.

→ xy → Makes it non linear

x^2

Autonomous Non-Linear Systems

In order to approach these systems, we want to analyze autonomous non-linear systems.

$$\left.\begin{aligned}\frac{dx}{dt} &= F(x,y)\\ \frac{dy}{dt} &= G(x,y)\end{aligned}\right]$$

No explicit dependence on t

→ Most physical systems satisfy this property.

. So we are ok with this restriction

Autonomous Equations to Systems

How did we handle autonomous first order equations? Can we extend that to autonomous first order systems?

- Phase lines for first order eqns.
  + Phase portraits for linear systems

→ Find Equilibrium Solutions

→ Use the actual function to determine if the solution was increasing or decreasing between them.

Equilibrium Solutions

The first step in this process is to find the equilibrium solutions to our system.

Equations $\frac{dy}{dt} = f(y)$

y-values where $f(y) = 0$.

System

$$\frac{dx}{dt} = F(x,y)$$

$$\frac{dy}{dt} = G(x,y)$$

Need points (x,y) so that both $F(x,y)$ and $G(x,y)$ are zero.

→ Factor the equations and see how the terms match up.

Example. Find all equilibrium solutions of the system

$$\frac{dx}{dt} = (x-2)(y-4)$$
$$\frac{dy}{dt} = (x-y)(y+3)$$

Need

$(x-2)(y-4) = 0$ AND $(x-y)(y+3) = 0$

$x = 2$ or $y = 4$

$x = y$

$y = -3$

$$\boxed{\cdot\ (2,2),\quad (2,-3),\quad (4,4)}$$

~~(2, 4), (-3, -3)~~