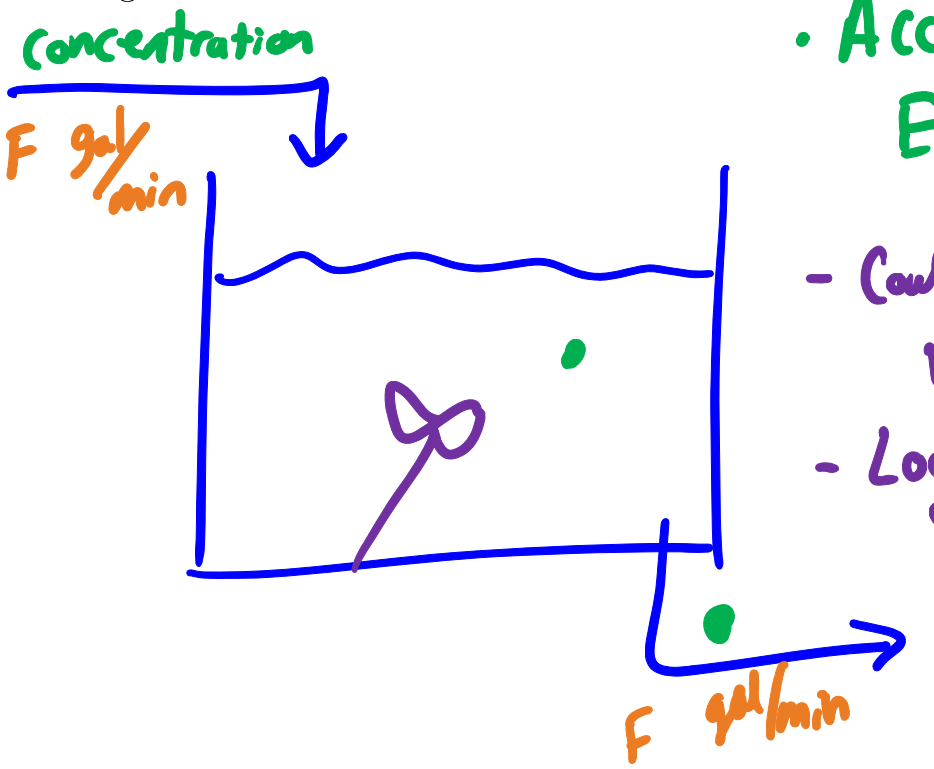
Tank Problems

One of the most common types of physical systems that can be modeled (and solved) using first-order equations is tank problems. The basic idea of these problems is that you have a tank of water and some concentration of salt (or any other substance) and has several pipes leading into or out of the tank that change the concentration of salt in the tank.

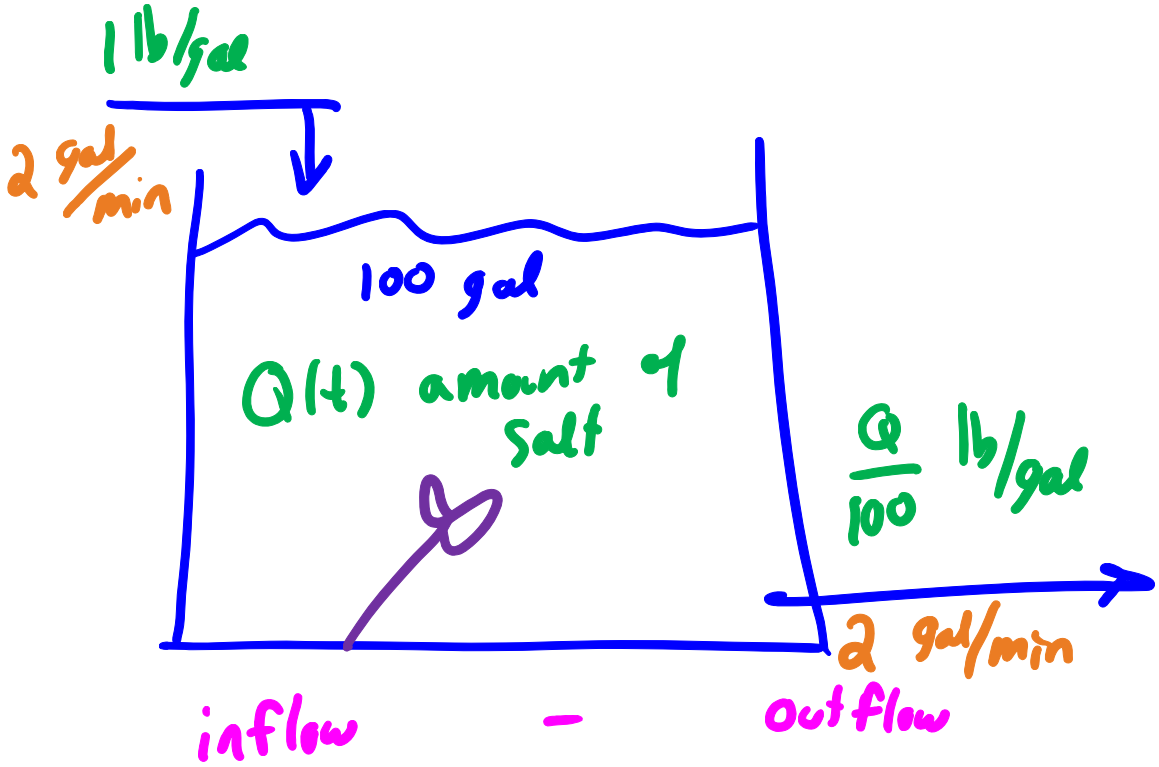


- Accumulation Equation.
  - Could look at volume
  - Look at Solute in the tank.

"Tank is Well-Mixed"
The concentration of solute in the out-flow stream is the same as that in the tank.

Example. A tank of water initially contains 100 gallons of water and 5 lbs of salt. A water stream containing 1 lb/gal of salt flows into the tank at a rate of 2 gal/min. The tank is well-mixed, and an exit stream removes water from the tank at a rate of 2 gal/min. Find a model for $Q(t)$, the amount of salt in the tank at any time t, and solve for $Q(t)$. What is the long-time behavior of Q, and does this make sense?

lbs



inflow − outflow

$$\frac{dQ}{dt}\ \frac{\text{lb}}{\text{min}} = 2\ \frac{\text{gal}}{\text{min}} \cdot \frac{1\ \text{lb}}{\text{gal}} - 2\ \frac{\text{gal}}{\text{min}} \cdot \frac{Q}{100}\ \frac{\text{lb}}{\text{gal}}$$

$$\frac{dQ}{dt} = 2 - \frac{Q}{50} \qquad Q(0) = 5$$

$$Q' + \frac{Q}{50} = 2 \qquad \mu(t) = e^{t/50}$$

$$\left(e^{t/50} Q' + \frac{1}{50} e^{t/50} Q\right) = 2e^{t/50}$$

$$e^{t/50} Q = 100\, e^{t/50} + C$$

$$Q(t) = 100 + C e^{-t/50}$$

$Q(0) = 5$ $\qquad Q(0) = 100 + C \rightarrow C = -95$

$$\boxed{Q(t) = 100 - 95 e^{-t/50}}$$

Long time behavior $\qquad Q(t) \to 100.$