Modeling with Differential Equations

Modeling with differential equations is the main way that the material in this course is applied in other fields, and it shows up everywhere.

If you want to model or calculate anything about stuff that's changing or moving, you need differential equations.

- Fluid flow in a pipe
- Pollutant concentration
- Price Fluctuation
- Disease Modeling.

- Learn to speak the language of differential equations to set up and solve these problems.

The Accumulation Equation

The general setup that underlies most of the models used in differential equations is the accumulation equation. This may also be called a balance equation (like a mass or energy balance) depending on the field.

$$\begin{bmatrix} \text{overall rate} \\ \text{of change} \end{bmatrix} = \begin{bmatrix} \text{rate being} \\ \text{added} \end{bmatrix} - \begin{bmatrix} \text{rate being} \\ \text{removed} \end{bmatrix}$$

$$\boxed{\text{total change}} = \text{rate in} - \text{rate out}$$

Derivative

many factors many factors

→ Comes from the physical system.

Ex Cup filling up with water

$$\frac{dV}{dt} = \text{hose}_1 + \text{hose}_2 - \text{bail}$$

We get more involved differential equations out of this concept when the rates of change depend on the value of the quantity itself.

Example. Let v represent the velocity of an object that is dropped. From physical principles, we know that the velocity will increase at a rate of $9.8\frac{m}{s^2}$ due to gravity. However, there is also drag on the object, causing it to be reduced at a rate of $0.1\frac{1}{m}$ multiplied by the square of the velocity. What is the differential equation that governs the velocity of this object?

$$\boxed{\frac{dv}{dt} = 9.8\frac{m}{s^2} - 0.1\frac{1}{m}\cdot v^2}$$

Net rate of change

Rate in

Rate out