Mechanical and Electrical Vibrations

Earlier in this chapter, we talked about physical situations that give rise to second order linear differential equations.

- Mass on a spring

$$mx'' + \gamma x' + kx = \begin{cases} 0 \\ F(t) \end{cases}$$

- RLC Circuits

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = \begin{cases} 0 \\ V(t) \end{cases}$$

$$\boxed{ay'' + by' + cy = f(t)}$$

$$f(t) = 0$$

$$a, b, c \geq 0$$

$$ay'' + by' + cy = 0 \text{ or } f(t)$$

Terminology

Most of our terminology for these equations comes from the mass-on-a-spring physical system.

- We say that the equation is unforced or free if the right-hand side is 0, i.e. if the equation is homogeneous.

- The b coefficient relates to drag force. This is called the damping coefficient.
  - If $b=0$, the equation is undamped.
  - If $b>0$, the equation is damped.

$$ay'' + by' + cy = 0$$

Types of Solutions

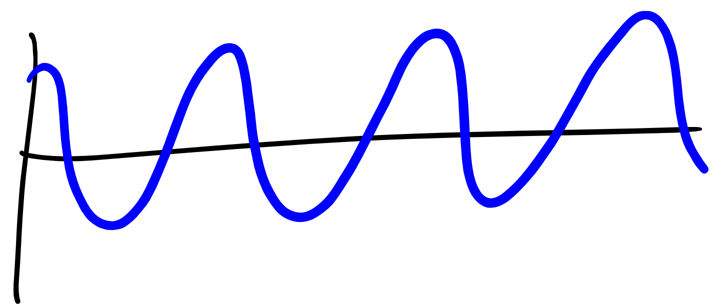
In the last few sections, we have worked out how to solve these equations. What can the solutions look like?

undamped

$b = 0$

$$ay'' + cy = 0$$

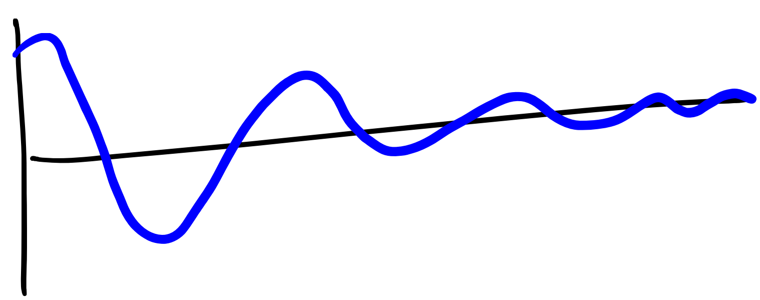
$$c_1 \sin(\omega t) + c_2 \cos(\omega t)$$



underdamped

$b^2 - 4ac < 0$ → complex solutions

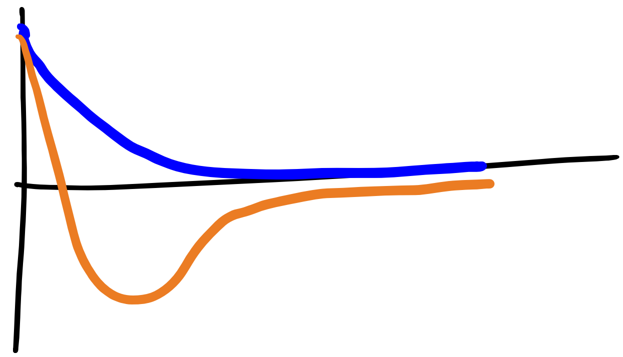
$$c_1 e^{-\lambda t} \cos(\mu t) + c_2 e^{-\lambda t} \sin(\mu t)$$



Critically damped

$b^2 - 4ac = 0$ → Repeated Root

$c_1 e^{rt} + c_2 t e^{rt}$



Overdamped

$b^2 - 4ac > 0$ → Two Real Roots.

$c_1 e^{r_1 t} + c_2 e^{r_2 t}$