Vectors

The main idea of linear algebra is a systematic approach to solving systems of equations. It has a large amount of interest and applications outside of that, but we'll be doing mostly the systems of equations here.

$$\begin{cases} 2x + 3y - z = 1 \\ x + 2y - 3z = 4 \\ 3x - y + 2z = 9 \end{cases}$$

Solution values of x, y, z that simultaneously solve all 3 equations.

or a point in 3 dimensions that satisfies this system.

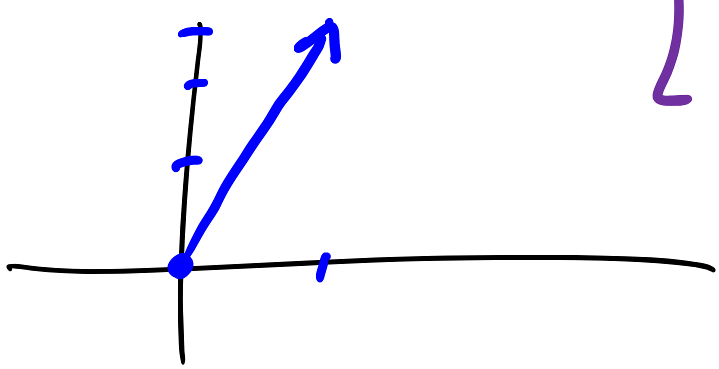
To do this correctly, we need some terminology to start with.

Definition 0.1. A *vector* is an n-tuple of numbers.

→ Similar to a point in the plane (2d) or in 3-space (3d)

Components → Individual numbers within each vector.

$(1,3)$



$\begin{bmatrix}1\\3\end{bmatrix}$

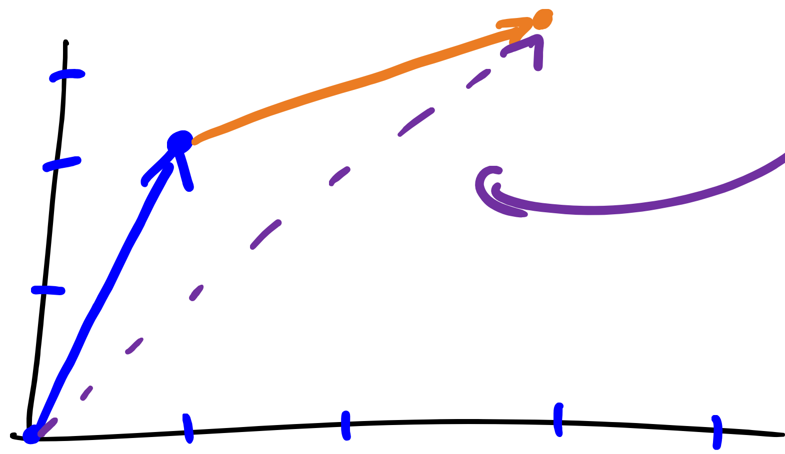
Writing Vectors → Column vectors

$\begin{bmatrix}1\\3\\0\\2\end{bmatrix}$ → Convenient notation for later.

What can we do with vectors?

There are two main types of operations we can perform on vectors. We can add them if they have the same number of components.

$$\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}1+2\\2+1\end{bmatrix}=\begin{bmatrix}3\\3\end{bmatrix}$$



$$\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}3\\4\\-1\end{bmatrix}$$

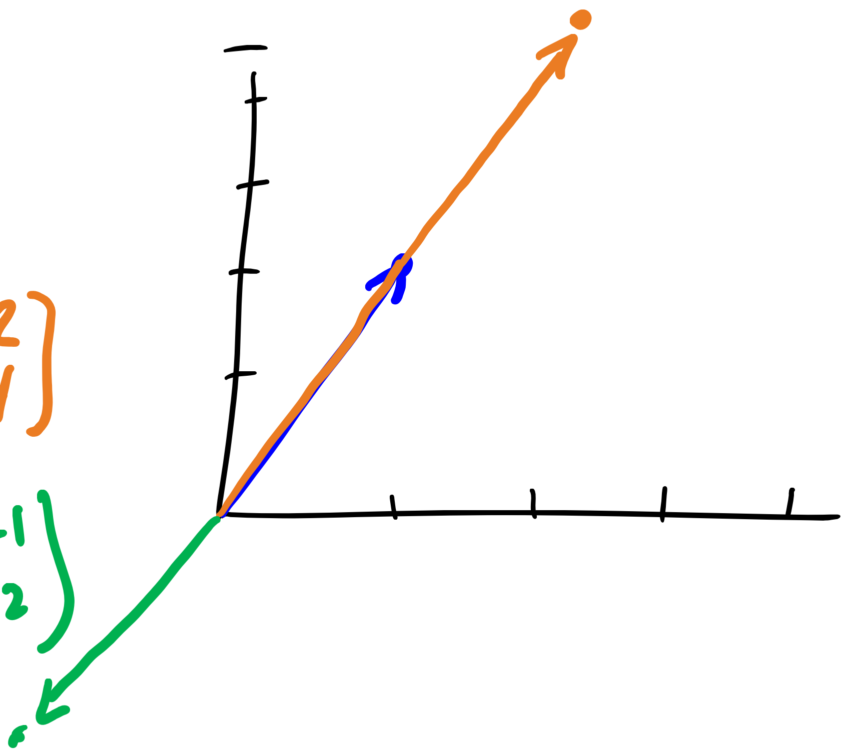
We can also multiply vectors by scalars.

- Extend the vector by the given factor

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$2\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$-1\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$



We can not multiply vectors. The only thing we have that is somewhat like multiplication is the dot product, but it takes two vectors and gives back a scalar, so it's not really a multiplication on vectors.

- Dot product

  $\vec{v} \cdot \vec{w} = a$

- Cross Product
  - → Only works in dimension 3.
  - → Not an option in general