

Systems of Equations

How do we go about solving systems of equations? The normal method for doing this is called *elimination*.

$$\begin{array}{r} 2x_1 + 3x_2 = 5 \\ 3 \left[\begin{array}{r} x_1 - x_2 = 0 \end{array} \right] \end{array}$$

Elimination \rightarrow Multiply 2nd Eqn by 3
then add the equations
together.

$$5x_1 = 5$$

$$x_1 = 1$$

$$x_2 = 1$$

Steps:

- 1) Swap the order of two equations.
- 2) Multiply an entire equation by a number.
- 3) Add some multiple of one equation to another.

Matrix Form

We can also write systems of equations in matrix form.

Example. Write the system of equations

$$3x_1 + 4x_2 = 6$$

$$x_1 - 5x_2 = 1$$

in matrix form.

$$\begin{bmatrix} 3x_1 + 4x_2 \\ x_1 - 5x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

Augmented Matrix

$[A|b]$

$$\begin{bmatrix} 3 & 4 & | & 6 \\ 1 & -5 & | & 1 \end{bmatrix}$$

Goal: Make it easy ³ to read off solutions

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array} \right]$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$x_1 = 2$$

$$x_2 = 3$$

→ Row Reduction
