Subspaces and Span

To get further into the idea of linear algebra, we need a few more definitions.

Definition 0.1. For a vector space V, a *subspace* W is a subset of V with the following properties:

Example. Let $V = \mathbb{R}^2$. Are the following subspaces of V?

1. $\mathbb{R}^2$
2. $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
3. $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} \text{ for all real numbers } a \right\}$
4. $\left\{ \begin{bmatrix} a \\ a \end{bmatrix}, \begin{bmatrix} a \\ -a \end{bmatrix} \text{ for all real numbers } a \right\}$

Another example of a subspace is the *span* of a set of vectors.

Definition 0.2. If $\vec{v}_1$, ..., $\vec{v}_n$ are vectors, then a *linear combination* of these vectors is

Definition 0.3. Let $\vec{v}_1, \vec{v}_2, ..., \vec{v}_m$ be any set of vectors in $\mathbb{R}^n$. The *span* of these vectors

Facts about Span

Questions about Spans

One of the most common questions to see about the span of a set of vectors is asking if a new vectors is in the span of a given set of vectors.

Example. Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the span of $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$?