Matrices

The next step up from vectors is matrices. What are these?

Definition 0.2. An $m \times n$ *matrix* is a rectangular array of mn numbers.

m rows , n columns

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 4 & 8 & 9 \\ 2 & 3 & -1 & 4 \\ 0 & 1 & 4 & 3 \end{bmatrix} \end{array}$$

3×4 matrix

Matrix A

A_{ij} = # in the i^{th} row and j^{th} column of A.

$A_{23} = -1$

$A_{14} = 9$

A vector with n components is just an $n \times 1$ matrix.

Operations on Matrices

What kinds of operations can we perform on matrices? Just like vectors, we can add them (provided they are the same size) and we can multiply by scalars.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 1 \end{bmatrix}$$

$$3\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix}$$

What about multiplying matrices? It turns out we do have a way of multiplying matrices, and it comes back to the idea of the dot product. For two matrices A and B, the ij entry of the product AB is the dot product of row i of A and column j of B.

$$A \qquad B \qquad = \qquad AB$$

$$m \times n \qquad p \times r \qquad m \times r$$

Need $n = p$ for this to work

$$\underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \cdot \underset{2\times 3}{\begin{bmatrix} 0 & 1 & 4 \\ 3 & -1 & 2 \end{bmatrix}} \rightarrow 2\times 3$$

$$= \begin{bmatrix} 1\cdot 0 + 2\cdot 3 & 1\cdot 1 + 2\cdot(-1) & 1\cdot 4 + 2\cdot 2 \\ 3\cdot 0 + 4\cdot 3 & 3\cdot 1 + 4(-1) & 3\cdot 4 + 4\cdot 2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 6 & -1 & 8 \\ 12 & -1 & 20 \end{bmatrix}}$$