There are many different spanning sets for $\mathbb{R}^2$, or any vector space/subspace.

$$\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\1\end{bmatrix}\right\} \quad \left\{\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}3\\2\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}\right\}$$

What is different about these spanning sets?

Linear Independence of Vectors

We have discussed linear independence of solutions to differential equations before. It was discussed as two solutions being "different enough" to meet every initial condition. The definition we use here is a little bit different, but it ends up being the same once all of the details are sorted out.

Definition 0.4. We say that vectors x_1, ..., x_n are *linearly independent* if

Example. Are the vectors $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ linearly independent? What about $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$?

From what we just saw, determining linear independence came down to trying to solve a system of equations, in particular, a homogeneous system of equations. This means we can use matrices and row reduction to try to work it out.

Example. Find a linearly independent subset of

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 5 \\ -10 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \\ -8 \\ 5 \end{bmatrix} \right\}$$