Finding Eigenvectors

Once we have the eigenvalue, we can then find the corresponding eigenvectors.

We found this λ

$$\det(A - \lambda I) = 0$$

$A\vec{v} = \lambda\vec{v}$

Now Find $\vec{v}$ so that $(A-\lambda I)\vec{v} = 0$

$\vec{v} \neq 0$

→ Characterize kernel of $A-\lambda I$.

- Row Reduce $A - \lambda I$
- Look at what the remaining equations tell us about what goes to zero.
  - → Should always get a zero row.
  - → At least one free variable.
    - → Can pick a non-zero value for it to get a specific eigenvector.

Example. Find the eigenvalues and corresponding eigenvectors for the matrix

$$A = \begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix}.$$

$$\det(A - \lambda I) = \det \begin{bmatrix} 4-\lambda & 3 \\ -2 & -1-\lambda \end{bmatrix}$$

$$= (4-\lambda)(-1-\lambda) + 6 = \lambda^2 - 3\lambda + 2$$

$$= (\lambda-1)(\lambda-2)$$

$\underline{\lambda = 1}$

$$A - I = \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \to v_1 + v_2 = 0$$

$$v_1 = -v_2$$

pick $v_2 = 1$

For $\lambda = 1$, vector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$x = 2$

$$A - 2I = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$$

$$2v_1 + 3v_2 = 0$$

$v_1 = -3 \qquad v_2 = 2$

$x = 2.$ vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 4 \end{bmatrix} = 2 \begin{bmatrix} -3 \\ 2 \end{bmatrix} \checkmark$$

Options for Eigenvalues and Eigenvectors

The eigenvalues of a matrix are the roots of a polynomial. This means there are only a few options for how they can behave.

Real polynomial

→ Real Roots

→ Complex conjugate pairs

Quadratics

→ can easily get explicitly.

→ Repeated roots can also cause problems.