Determinants

For a square matrix A, the determinant is a number that gives a characterization of how this matrix behaves. Let's start with 2x2 matrices.

Definition 0.1. For a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant

Larger Matrices

How do we compute the determinant of bigger matrices? We do so recursively.

Definition 0.2. For an $n \times n$ square matrix A, we defined the *ij minor* of A, denoted A_{ij}, to be

Example. What are A_{23} and A_{41} for the matrix

$$A = \begin{bmatrix} 1 & 0 & 5 & -1 \\ 3 & 6 & -2 & 1 \\ 2 & 3 & 0 & 1 \\ 1 & 1 & 9 & 3 \end{bmatrix}?$$

With this defined, we can now compute the determinant of larger matrices.

Definition 0.3. For an $n \times n$ square matrix A, the determinant $\det(A)$ is computed by choosing a row i between 1 and n, and computing

$$\det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij})$$

where a_{ij} is the entry in row i, column j of the matrix A, and A_{ij} is the ij minor of A. It can also be computed by choosing a column j between 1 and n and computing

$$\det(A) = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij}).$$

Example. Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 0 \\ 3 & 1 & 0 \end{bmatrix}.$$