

# General Solutions and Initial Value Problems

Suppose that I gave you the differential equation

$$\frac{dy}{dt} = 2t. \quad y(t)$$

What is the solution?

$y(t) = t^2$  is a solution

So is  $y(t) = t^2 + 1$

$y(t) = t^2 + C$  will be a solution for any constant  $C$ .

A general solution to a differential equation is one with an arbitrary constant in it

To determine  $C$ , need more information

$\frac{dy}{dt} = 2t$  and  $y(1) = 2$  Initial Condition

- Initial Value Problem •

**Example.** Verify that  $\phi(t) = e^{2t} + C$  is not the general solution to the equation  $\frac{dy}{dt} = 2y$ , but  $y(t) = Ce^{2t}$  works. What is the value of  $C$  so that this function satisfies the initial condition  $y(0) = 4$

$$\phi(t) = e^{2t} + C$$

$$\phi'(t) = 2e^{2t} \neq 2\phi(t) \quad \times$$

$$y(t) = Ce^{2t}$$

$$y'(t) = 2Ce^{2t} = 2y \quad \checkmark$$

$$y(0) = Ce^{2 \cdot 0} = C = 4$$

Pick  $C = 4$ .

$$y(t) = 4e^{2t}$$