Higher Order Equations

We can also consider higher order equations and attempt to analyze and solve them. Just like going from first to second order added a fair bit of complexity, so does stepping up to higher orders.

General n^{th} order equation

$$y^{(n)} = F(t, y, y', y'', \ldots, y^{(n-1)})$$

→ Generally no hope of solving this.

n^{th} order linear equation

$$y^{(n)} + f_{n-1}(t)y^{(n-1)} + f_{n-2}(t)y^{(n-2)} + \cdots + f_1(t)y' + f_0(t)y = g(t)$$

If $f_0, f_1, \ldots, f_{n-1}$ are all constants, then constant coefficient.

Homogeneous if $g(t) = 0$

Non-Homogeneous if $g(t) \neq 0$.

Initial Value Problems

Need n initial conditions to fully specify the problem.

Given in the form

$$y(t_0) = y_0 \qquad y'(t_0) = y_1, \; \ldots$$

$$y^{(n-1)}(t_0) = y_{n-1}$$

How can we solve them?

The process for solving these is similar to what we had for second order.

If not constant coefficient, really no chance.

If constant coefficient:

- Guess e^{rt} as a solution
- Get characteristic equation
- Find the roots. (No Quad. Formula)
- Handle like before.
  - → Can have more complicated structures.

For Non-Homogeneous

→ Undetermined Coefficients still works.

→ Extension of variation of parameters

→ Much More Complicated.

Example. Find the general solution to the differential equation

$$y''' + 2y'' - 4y' - 8y = 0.$$

Guess e^{rt} as a solution

Characteristic Equation $r^3 + 2r^2 - 4r - 8 = 0$

$r^2(r+2) \quad -4(r+2)$

$(r^2-4)(r+2) = (r-2)(r+2)(r+2)$

Root at 2, Double root at -2.

General Solution

$$\boxed{y(t) = C_1 e^{2t} + C_2 e^{-2t} + C_3 \underline{te}^{-2t}}$$