First-Order Linear Equations

The next type of equation we want to try to solve is first-order linear equations. We mentioned before that linear equations made things easier, but we haven't looked into how to solve them yet.

$$a(t)\,y' + b(t)\,y = c(t)$$

Standard Form

$$\boxed{y' + p(t)\,y = g(t)}$$

Definition. Assume that we have a differential equation of the form $\frac{dy}{dt} + p(t)y = g(t)$. We say that this equation is

- Constant coefficient if the function $p(t)$ is a constant [independent of t]

- homogeneous if $g(t) = 0$

and it is non-homogeneous otherwise.

If $g(t) = 0$, then this equation is separable, and we know how to solve those. If not, then what can we do?

Example. Solve the differential equation

$$\underbrace{t^2 \frac{dy}{dt} + 2ty}_{} = e^t.$$

$$(f(t)g(t))' = f'(t)\,g(t) + f(t)\,g'(t)$$

$$(t^2 y(t))' = t^2 \frac{dy}{dt} + 2ty$$

$$\int (t^2 y(t))'\,dt = \int e^t\,dt$$

$$t^2 y(t) = e^t + C$$

$$\boxed{y(t) = \frac{e^t}{t^2} + \frac{C}{t^2}}$$