Autonomous Equations

Definition. We say that a first order differential equation $y' = f(t, y)$ is *autonomous* if the function $f(t,y)$ is independent of t, that is, is only a function of y.

- Equation has no direct dependence on the value of t.
  - → Population and population growth.
    - → Bacteria growth
    - → Some populations have more complicated growth but these models will focus on the simple ones.

We have already solved some autonomous equations previously.

Example. Solve the differential equation

$$\frac{dy}{dt} = ky \qquad y(0) = y_0$$

- Can solve either using Separable equations or first order linear methods

$y(t) = y_0 e^{kt}$ → Exponential growth model.

Equilibrium Solutions

The simple autonomous equations can be solved fairly directly. However, there are many more complicated autonomous equations for which we can not write out a specific (explicit) solution.

Example. Analyze the solutions to the differential equation

1 1-5 (1+3)² <0

$$\frac{dy}{dt} = y(y-5)(y+3)^2$$

for a variety of initial conditions.

- Could try to solve as a separable equation.
  → No way to write an explicit solution

What are some simple solutions to this equation?

$y(t) = 0$ $\quad y' = 0 = 0(0-5)(0+3)^2$ ✓

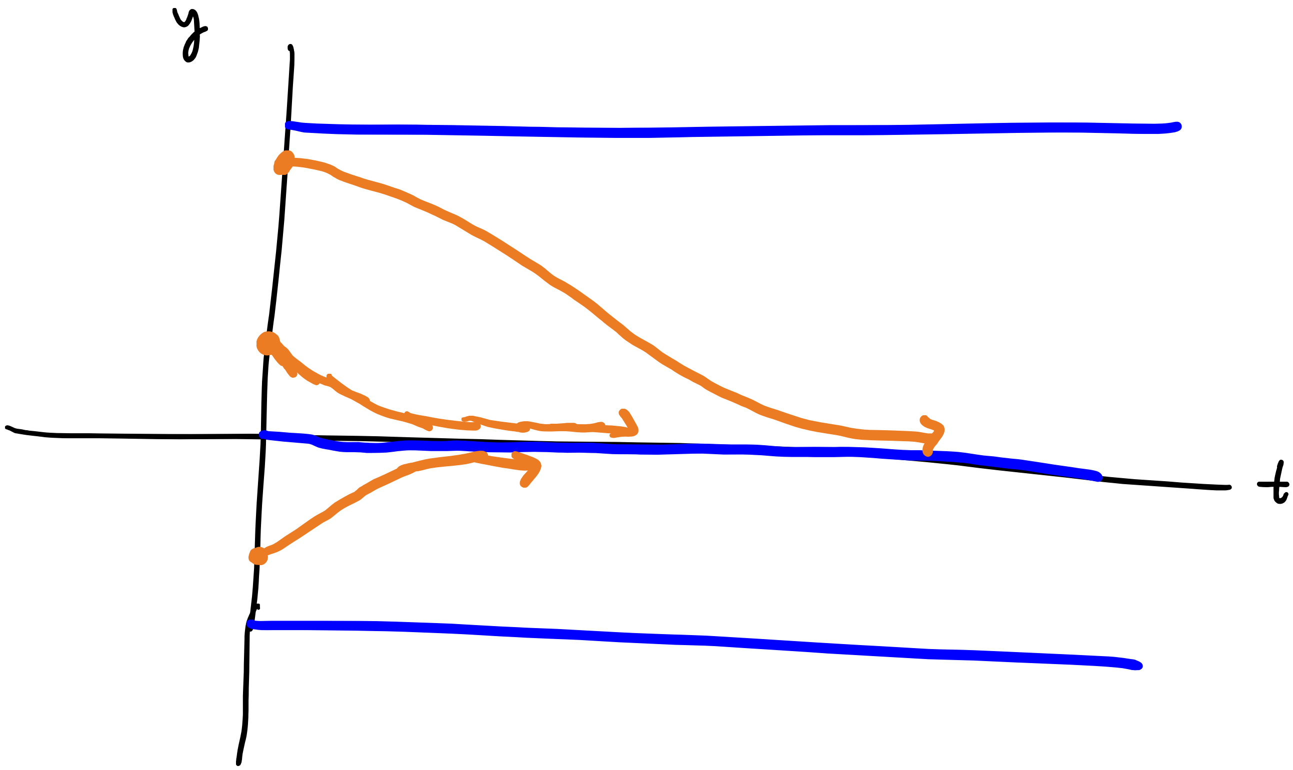
$y(t) = 5$

$y(t) = -3$

These are equilibrium solutions to this problem.

- Based on this equation, I know that the Existence and Uniqueness Thms apply.

→ My solution curves can not cross!



Definition. A *phase line* for the autonomous differential equation $\frac{dy}{dt} = f(y)$ is a visual representation of for which y values the solution will be increasing or decreasing based on the sign of f.

Example. Analyze the solutions to the differential equation

$$\frac{dy}{dt} = y(y-5)(y+3)^2$$

for a variety of initial conditions.