

Autonomous Equations

Definition. We say that a first order differential equation $y' = f(t, y)$ is *autonomous* if

We have already solved some autonomous equations previously.

Example. Solve the differential equation

$$\frac{dy}{dt} = ky \quad y(0) = y_0$$

Equilibrium Solutions

The simple autonomous equations can be solved fairly directly. However, there are many more complicated autonomous equations for which we can not write out a specific (explicit) solution.

Example. Analyze the solutions to the differential equation

$$\frac{dy}{dt} = y(y - 5)(y + 3)^2$$

for a variety of initial conditions.

Definition. A *phase line* for the autonomous differential equation $\frac{dy}{dt} = f(y)$ is

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