Euler's Method

The fact that solutions exist doesn't tell us anything about how to find them. There are also many differential equations that we don't know how to solve. The main solution to this is numerical methods. These allow us to numerically approximate the solution to a differential equation without needing to know the solution analytically.

- This gets an approximation.
  - Hopefully we can get one that is close enough.
- Still need existence and uniqueness.
  - No point looking for a solution if it is not there.

The standard (and easiest) way to numerically solve differential equations is Euler's Method.

$$\frac{dy}{dt} = f(t, y) \quad \Big] \text{ Slope of the tangent line.}$$

- Tangent line = best linear approximation
  - → So follow the tangent line for a short distance.
- Take this new point and repeat the process.

Method

Assume that we have a differential equation $y' = f(t, y)$. How do we approximate the solution by Euler's Method?

1. Start at (t_0, y_0). Pick a step size h.

2. Slope of the tangent line is $f(t_0, y_0)$

3. New point $t_1 = t_0 + h$

$$y_1 = y_0 + f(t_0, y_0)h$$

This gives the next point (t_1, y_1)

4. Repeat process to get $(t_2, y_2)\ldots$

5. Continue until reaching a final time