Combining Sine and Cosine Functions

A combination of the form

$$A\sin(\omega t) + B\cos(\omega t)$$

looks like a single oscillation if you draw the graph. This is because this can always be written in the form

$$R\cos(\omega t - \phi)$$

for some value of R and ϕ.

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$R\cos(\omega t - \phi) = R\left[\cos(\omega t)\cos(\phi) + \sin(\omega t)\sin(\phi)\right]$$

$$= R\cos\phi \cos(\omega t) + R\sin\phi \sin(\omega t)$$

$$= A \cos(\omega t) + B \sin(\omega t)$$

$$\boxed{A = R\cos\phi \qquad B = R\sin\phi}$$

$$A^2 + B^2 = R^2\cos^2\phi + R^2\sin^2\phi = R^2\underbrace{(\cos^2\phi + \sin^2\phi)}_{1}$$

$$\boxed{R = \sqrt{A^2+B^2}}$$

$$\frac{B}{A} = \frac{R\sin\phi}{R\cos\phi} = \tan(\phi)$$

$$\boxed{\phi = \tan^{-1}(B/A)}$$

Connecting to Second Order Equations

How do all of these quantities relate to the initial conditions given in a problem?

$$y(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$y(0) = A$$

$$y'(0) = \omega B \rightarrow B = \frac{y'(0)}{\omega}$$

$$R = \sqrt{A^2 + B^2}$$

$$= \boxed{\sqrt{(y(0))^2 + \frac{y'(0)^2}{\omega^2}}}$$