MATH 251: Quiz 7

December 10, 2015

Name: Solutions Sec: ____________

1. Let $c(t) = \langle 3\sin(t), 4t, 3\cos(t)\rangle$ be a curve, over the range $0 \le t \le 2\pi$. For the vector field $\vec{F}(x,y,z) = \langle z, x^2+y^2, 1\rangle$ and the function $f(x,y,z) = y^2+z^2$, compute

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$$\int_{c(t)} \vec{F}\cdot d\vec{s} \quad \text{and} \quad \int_{c(t)} f\, ds$$

$$c'(t) = \langle 3\cos(t), 4, -3\sin(t)\rangle \qquad \|c'(t)\| = \sqrt{9\cos^2 t + 16 + 9\sin^2 t} = 5$$

$$\int_{c(t)} \vec{F}\cdot d\vec{s} = \int_0^{2\pi} \langle 3\cos(t), 9\sin^2 t + 16t^2, 1\rangle \cdot \langle 3\cos(t), 4, -3\sin(t)\rangle\, dt$$

$$= \int_0^{2\pi} 9\cos^2 t + 36\sin^2 t + 64t^2 - 3\sin(t)\, dt \quad (2)$$

$$= \int_0^{2\pi} 9 + 27\underbrace{\sin^2 t}_{\frac{1}{2}-\frac{1}{2}\cos 2t} + 64t^2 - 3\sin(t)\, dt$$

$$= 9t + \frac{27}{2}t - \frac{27}{4}\sin 2t + \frac{64}{3}t^3 + 3\cos(t)\Big|_0^{2\pi} \quad (1)$$

$$= 18\pi + 27\pi - 0 + \frac{64\cdot 8}{3}\pi^3 + 0 = \boxed{45\pi + \frac{512}{3}\pi^3}$$

$$\int_{c(t)} f\, ds = \int_0^{2\pi} (16t^2 + 9\cos^2 t)5\, dt = 5\int_0^{2\pi} 16t^2 + 9\left(\frac{1}{2} + \frac{1}{2}\cos 2t\right) dt \quad (2)$$

$$= 5\left[\frac{16}{3}t^3 + \frac{9}{2}t + \frac{9}{4}\sin 2t\right]\Big|_0^{2\pi}$$

$$= 5\left[\frac{128}{3}\pi^3 + 9\pi + 0\right] = \boxed{\frac{640}{3}\pi^3 + 45\pi} \quad (1)$$

2. Determine if the vector field $\vec{F} = \langle 2xyz + yze^{xy}, x^2z + xze^{xy} + 2y, x^2y + e^{xy} \rangle$ is conservative, and if so, find a potential function for $\vec{F}$.

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$$\int F_x \, dx = x^2yz + ze^{xy} + f_1(y,z) \quad (1)$$

$$\int F_y \, dy = x^2yz + ze^{xy} + y^2 + f_2(x,z) \quad (1)$$

$$\int F_z \, dz = x^2yz + ze^{xy} + f_3(x,y) \quad (1)$$

$$\Rightarrow \boxed{V = x^2yz + ze^{xy} + y^2 + C} \quad (1)$$