MATH 251: Quiz 6

November 19, 2015

Name: Solutions Sec: ____________

1. Integrate the function $f(x, y, z) = x^2 + 3yz$ over the rectangular prism $0 \le x \le 3$, $1 \le y \le 4$, $1 \le z \le 3$.

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$$\int_0^3\int_1^4\int_1^3 x^2+3yz\,dz\,dy\,dx \quad (1)$$

$$= \int_0^3\int_1^4 x^2z + \tfrac{3}{2}yz^2\Big|_1^3\,dy\,dx$$

$$= \int_0^3\int_1^4 2x^2+12y\,dy\,dx \quad (1)$$

$$= \int_0^3 2x^2y+6y^2\Big|_1^4\,dx = \int_0^3 6x^2+90\,dx$$

$$= 2x^3+90x\Big|_0^3 = 54+270 = \boxed{324} \quad (1)$$

2. Find the volume of the region inside the cylinder $x^2 + y^2 = 9$ between the xy-plane and the plane $x + y + z = 5$. [Use cylindrical coordinates]

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$$\Rightarrow \int_0^{2\pi}\int_0^3\int_0^{5-x-y\,=\,5-r\cos\theta-r\sin\theta} 1\cdot r\,dz\,dr\,d\theta \quad (2)$$

$$= \int_0^{2\pi}\int_0^3 rz\Big|_0^{5-r\cos\theta-r\sin\theta}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^3 5r - r^2\cos\theta - r^2\sin\theta\,dr\,d\theta \quad (1)$$

$$= \int_0^{2\pi} \tfrac{5}{2}r^2 - \tfrac{r^3}{3}\cos\theta - \tfrac{r^3}{3}\sin\theta\Big|_0^3\,d\theta = \int_0^{2\pi}\tfrac{45}{2} - 9\cos\theta - 9\sin\theta\,d\theta$$

$$= \tfrac{45}{2}\theta - 9\sin\theta + 9\cos\theta\Big|_0^{2\pi}$$

$$= 45\pi \quad (1)$$

3. Integrate the function $g(x, y, z) = z$ over the upper hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$. [Use spherical coordinates]

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$0 \leq \rho \leq 4 \qquad 0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi/2$

$$\int_0^4 \int_0^{2\pi} \int_0^{\pi/2} (\rho \cos\varphi)\rho^2 \sin\varphi \, d\varphi \, d\theta \, d\rho \quad (1)$$

$$= \int_0^4 \int_0^{2\pi} \int_0^{\pi/2} \rho^3 \sin\varphi \cos\varphi \, d\varphi \, d\theta \, d\rho$$

$$= \int_0^4 \int_0^{2\pi} \rho^3 \frac{\sin^2\varphi}{2} \Big|_0^{\pi/2} d\theta \, d\rho \quad (1)$$

$$= \frac{1}{2} \int_0^4 \int_0^{2\pi} \rho^3 \, d\theta \, d\rho$$

$$= \frac{1}{2} \int_0^4 \rho^3 \theta \Big|_0^{2\pi} d\rho = \pi \int_0^4 \rho^3 d\rho$$

$$= \pi \frac{\rho^4}{4} \Big|_0^4 = 64\pi \quad (1)$$