MATH 251: Quiz 5

November 5, 2015

Name: Solutions Sec: ____________

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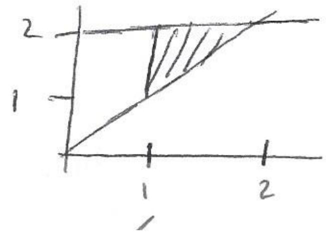
1. Compute the following integrals.

$$\int_0^3 \int_1^2 x^2 + 4xy^2 + e^y \; dx \; dy$$

$$\int_1^2 \int_x^2 y \ln(xy) \; dy \; dx$$



$$\int_0^3 \int_1^2 x^2 + 4xy^2 + e^y \, dx \, dy$$

$$= \int_0^3 \frac{x^3}{3} + 2x^2y^2 + xe^y \Big|_1^2 \, dy \quad (1)$$

$$= \int_0^3 \frac{8}{3} + 8y^2 + 2e^y - \frac{1}{3} - 2y^2 - e^y \, dy$$

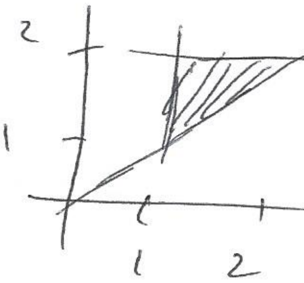
$$= \int_0^3 \frac{7}{3} + 6y^2 + e^y \, dy$$

$$= \frac{7}{3}y + 2y^3 + e^y \Big|_0^3 \quad (1)$$

$$= 7 + 54 + e^3 - 0 - 0 - 1$$

$$= \boxed{60 + e^3} \quad (1)$$

$$\int_1^2 \int_x^2 y \ln(xy)\, dy\, dx = \int_1^2 \int_1^y y \ln(xy)\, dx\, dy$$



$u = xy \quad du = y\,dx$

$$= \int_1^2 \int_y^{y^2} \ln(u)\, du\, dy$$

$$= \int_1^2 u \ln u - u \Big|_y^{y^2} dy$$

$$= \int_1^2 y^2 \ln y^2 - y^2 - y \ln y + y \, dy$$

$$= \int_1^2 (2y^2 - y) \ln y + y - y^2 \, dy$$

Integration by parts

$u = \ln y \qquad v = \frac{2}{3}y^3 - \frac{y^2}{2}$

$du = \frac{1}{y}\,dy \qquad dv = (2y^2 - y)\,dy$

So $\int (2y^2 - y)\ln y \, dy = \left(\frac{2}{3}y^3 - \frac{y^2}{2}\right)\ln(y) - \int \frac{2}{3}y^2 - \frac{y}{2}\, dy$

$$= \left(\frac{2}{3}y^3 - \frac{y^2}{2}\right)\ln y - \frac{2}{9}y^3 + \frac{y^2}{4}$$

Therefore

$$= \left(\frac{2}{3}y^3 - \frac{y^2}{2}\right)\ln(y) - \frac{2}{9}y^3 + \frac{y^2}{4} + \frac{y^2}{2} - \frac{y^3}{3} \Bigg|_1^2$$

$$= \left(\frac{16}{3} - 2\right)\ln(2) - \frac{16}{9} + 1 + 2 - \frac{8}{3} - \left(0 - \frac{2}{9} + \frac{1}{4} + \frac{1}{2} - \frac{1}{3}\right) = \boxed{\frac{10}{3}\ln(2) - \frac{59}{36}}$$

$$\int_1^2 \int_x^2 y \ln(xy)\, dy\, dx \qquad \begin{array}{ll} u = \ln(xy) & v = y^2/2 \\ du = 1/y\, dy & dv = y\, dy \end{array}$$

Integration by parts:

$$= \int_1^2 \left(\frac{y^2}{2} \ln(xy) \Big|_x^2 - \frac{1}{2} \int_x^2 y\, dy \right) dx$$

$$= \int_1^2 \frac{y^2}{2} \ln(xy) - \frac{y^2}{4} \Big|_x^2 \, dx$$

$$= \int_1^2 2 \ln 2x - 1 - \frac{x^2}{2} \ln(x^2) + \frac{x^2}{4} \, dx$$

$$= \int_1^2 2 \ln 2x - x^2 \ln x + \frac{x^2}{4} - 1 \, dx$$

$$u = 2x$$
$$du = 2dx$$
$$= \int \ln u\, du$$

$$\int x^2 \ln x = \frac{x^3}{3} \ln x - \int \frac{x^2}{3} dx$$

$$u = \ln x \qquad du = x^3/3$$
$$du = \frac{1}{x} dx \qquad dv = x^2\, dx$$

So combining these

$$= \left[2x \ln(2x) - 2x - \frac{x^3}{3} \ln x + \frac{x^3}{9} + \frac{x^3}{12} - x \right] \Big|_1^2$$

$$= 4 \ln 4 - 4 - \frac{8}{3} \ln 2 + \frac{8}{9} + \frac{8}{12} - 2$$
$$- \left(2 \ln 2 - 2 - 0 + \frac{1}{9} + \frac{1}{12} - 1 \right)$$

$$= 6 \ln 2 - 2 - \frac{8}{3} \ln 2 + \frac{7}{9} + \frac{7}{12} - 1 = \boxed{\frac{10}{3} \ln(2) - \frac{59}{36}}$$

2. Find the area between the curves $y = 2x$ and $y = x^2$. This is the area below $y = 2x$ and above $y = x^2$.

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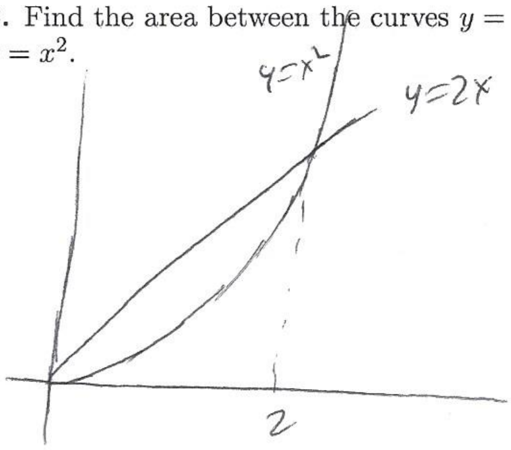

$$x^2 = 2x$$
$$x = 0, \quad x = 2$$

(2)

$$\int_0^2 \int_{x^2}^{2x} 1 \, dy \, dx$$

$$= \int_0^2 y \Big|_{x^2}^{2x} dx = \int_0^2 2x - x^2 \, dx$$

$$= x^2 - \frac{x^3}{3} \Big|_0^2$$

(2)

$$= 4 - \frac{8}{3} = \boxed{4/3}$$